INTERNATIONAL STANDARD

ISO 3951-1

First edition
2005-04-01

Sampling procedures for inspection by variables —

Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

Règles d'échantillonnage pour les contrôles par mesures —

Partie 1: Spécifications pour les plans d'échantillonnage simples indexés d'après le niveau de qualité acceptable (NQA) pour le contrôle lot par lot pour une caractéristique de qualité unique et un NQA unique

Reference number
ISO 3951-1:2005(E)

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Published in Switzerland

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-1 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This first edition of ISO 3951-1 cancels and replaces ISO 3951:1989, of which it constitutes a technical revision. The most significant differences between ISO 3951-1:2003 and ISO 3951:1989 are as follows.

- The acronym AQL now stands for Acceptance Quality Limit rather than Acceptable Quality Level, in order to reflect more accurately its function.
- The coverage of this part of ISO 3951 is constrained to a single, normally distributed variable with a single class of nonconformity. This part of ISO 3951 includes the case of combined control of double specification limits, but procedures for separate or complex control of double specification limits are deferred to ISO 3951-2. More general procedures that can be used for multiple characteristics and/or multiple AQLs are also given in ISO 3951-2.
- The plans have been modified so that their operating characteristic curves more closely match those of the plans in ISO 2859-1. The sample sizes for both the "s" method and the "σ" method are constant along rows of the master tables.
- All acceptability constants (see Annexes B, C, G and I) have been revised and tabulated to three decimal places for an extended range of AQLs corresponding to ISO 2859-1:1999.
- All tabulated values of operating characteristics have been recalculated and related directly to reduced inspection as well as to normal and tightened inspection.
- The annex containing the general statistical theory has been removed. It is planned ultimately to reintroduce this within a guidance document to sampling procedures for inspection by variables.
- Tables that are required for implementing the procedures have been relocated into annexes.
- The annex dealing with the "R" method has been eliminated, now that the availability of calculators with a standard deviation function key is so widespread. Data for acceptance sampling by variables is often substantially more expensive to acquire than data for sampling by attributes, and the "s" method makes more efficient use of this data.

ISO 3951 currently consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

— *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

— *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

The following parts are under preparation:

— *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

— *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables*

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the Acceptance Quality Limit (AQL), and is designed for users who have simple requirements. A more comprehensive and technical treatment is given in ISO 3951-2. This part of ISO 3951 is complementary to ISO 2859-1.

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of an acceptable quality have a high probability of acceptance, and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide:

a) automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;

b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) if consistently good quality is achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is intended for application to a single quality characteristic that is measurable on a continuous scale. For two or more such quality characteristics, see ISO 3951-2.

It is assumed in this part of ISO 3951 that measurement error is negligible. For information on allowing for measurement error, see Reference [17] in the Bibliography.

For double specification limits, this part of ISO 3951 treats combined control. For other types of control, see ISO 3951-2.

Inspection by variables for percent nonconforming items, as described in the present document, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

— unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;

— a single specification limit, or combined control of double specification limits;

— normal inspection, tightened inspection or reduced inspection.

Fourteen annexes are provided. Annexes A to I provide the tables needed to support the procedures. Annex J indicates how the sample standard deviation, "s", and the presumed known value of the process standard deviation, "σ", should be determined. Annex K provides the statistical theory underlying the calculation of the consumer's risks, together with tables showing these risks for normal, tightened and reduced inspection as well as for the "s" and "σ" methods. Annex L provides similar information for the producer's risks. Annex M gives the general formula for the operating characteristic of the "σ" method. Annex N provides the statistical theory underlying the estimation of the process fraction nonconforming under the "s" method for sample sizes 3 and 4, which for technical reasons are treated differently from the other sample sizes in this part of ISO 3951.

Table 1 is intended to facilitate the use of this part of ISO 3951 by directing users to the paragraphs and tables concerning any situation with which they may be confronted. Table 1 only deals with Clauses 15, 16, 20, 21 and 22; in every case, it is necessary to have first read the other clauses.

Table 1 — Summary table

Inspection type	Single specification limit						Double specification limits with combined control					
	"s" method			"σ" method			"s" method			"σ" method		
	Clauses or subclauses	Tables	Charts	Clauses or subclauses	Tables	Charts	Clauses or subclauses	Tables	Charts	Clauses or subclauses	Tables	Charts
Normal inspection	15.1, 15.2, 15.3 and 20.1	A.1, A.2, B.1, B to R	B to R	16.1, 16.2 and 20.1	A.1, A.2, C.1, B to R *	B to R *	15.1, 15.4 and 20.1	A.1, A.2, D.1, F (for n = 3), G (for n = 3 or 4), B to R*	*s*-D to *s*-R, B to R *	16.1, 16.3 and 20.1	A.1,A.2, C.1, E, B to R *	B to R *
Switching between normal and tightened inspection	20.2, 20.3	B.1, B.2	B to R	20.2, 20.3	C.1, C.2	B to R *	20.2, 20.3	D.1, D.2	*s*-D to *s*-R, B to R *	20.2, 20.3	C.1, C.2, E	B to R *
Switching between normal and reduced inspection	20.4, 20.5	B.1, B.3	B to R	20.4, 20.5	C.1, C.3	B to R *	20.4, 20.5	D.1, D.3	*s*-D to *s*-R, B to R *	20.4, 20.5	E	B to R *
Switching between tightened and dis-continued inspection	21	B.2	B to R	21	C.2	B to R *	21	D.2	*s*-D to *s*-R, B to R *	21	E	B to R *
Switching between the "s" and "σ" methods	22	Annex K		22	Annex K		22	Annex K		22	Annex K	

* But see 8.4.

Sampling procedures for inspection by variables —

Part 1:
Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

CAUTION —The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

1 Scope

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables, in which the acceptability of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is primarily designed for application under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process;

b) where only a single quality characteristic x of these products is taken into consideration, which must be measurable on a continuous scale;

c) where the measurement error is negligible, i.e. with a standard deviation no more than 10 % of the process standard deviation;

d) where production is stable (under statistical control) and the quality characteristic x is distributed according to a normal distribution or a close approximation to the normal distribution;

e) where a contract or standard defines an upper specification limit U, a lower specification limit L, or both; an item is qualified as conforming if and only if its measured quality characteristic x satisfies the appropriate one of the following inequalities:

   1) $x \geqslant L$ (i.e. the lower specification limit is not violated);

   2) $x \leqslant U$ (i.e. the upper specification limit is not violated);

   3) $x \geqslant L$ and $x \leqslant U$ (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are called cases with a single specification limit, and 3) a case with double specification limits.

If double specification limits apply, it is assumed in this part of ISO 3951 that conformance to both specification limits is equally important to the integrity of the product; in such cases it is appropriate to apply a single AQL to the combined percentage of product outside the two specification limits. This is referred to as combined control.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and statistical terms*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1, and ISO 3534-2 apply.

3.1
inspection by variables
inspection by measuring the magnitude of a characteristic of an item

[ISO 3534-2]

3.2
sampling inspection
inspection of selected items in the group under consideration

[ISO 3534-2]

3.3
acceptance sampling inspection
acceptance sampling
sampling inspection (3.2) to determine whether or not to accept a lot or other amount of product, material or service

[ISO 3534-2]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on a specified quality characteristic of each item in a sample from a lot

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process, expressed as a proportion

3.6
acceptance quality limit (AQL)
worst tolerable **process fraction nonconforming** (3.5) when a continuing series of lots is submitted for **acceptance sampling** (3.3)

NOTE See Clause 5.

3.7
quality level
quality expressed as a rate of occurrence of nonconforming units

3.8
limiting quality
LQ
quality level (3.7), when a lot is considered in isolation, which, for the purposes of **acceptance sampling inspection** (3.3), is limited to a low probability of acceptance

[ISO 3534-2]

NOTE 1 In this part of ISO 3951, the probability of acceptance is limited to 10 %.

NOTE 2 See 13.1.

3.9
nonconformity
nonfulfilment of a requirement

[ISO 9000]

3.10
nonconforming unit
unit with one or more nonconformities

[ISO 3534-2]

3.11
"s" **method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the sample standard deviation

[ISO 3534-2]

NOTE See Clause 15.

3.12
"σ" **method acceptance sampling plan**
acceptance sampling (3.3) plan by variables using the presumed value of the process standard deviation

[ISO 3534-2]

NOTE See Clause 16.

3.13
specification limit
conformance boundary specified for a characteristic

[ISO 3534-2]

3.14
lower specification limit
L
specification limit [3.13] that defines the lower conformance boundary

[ISO 3534-2]

3.15
upper specification limit
U
specification limit (3.13) that defines the upper conformance boundary

[ISO 3534-2]

3.16
combined control
requirement when both upper and lower limits are specified for the quality characteristic and an **AQL** (3.6) is given that applies to the combined percent nonconforming beyond the two limits

NOTE 1 See 5.3.

NOTE 2 The use of combined control implies that nonconformity beyond either **specification limit** (3.13) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.17
acceptability constant
k
constant depending on the specified value of the **acceptance quality limit** (3.6) and the sample size, used in the criteria for accepting the lot in an **acceptance sampling** (3.3) plan by variables

[ISO 3534-2]

NOTE See 15.2 and 16.2.

3.18
quality statistic
Q
function of the **specification limit** (3.13), the sample mean, and the sample or process standard deviation, used in assessing the acceptability of a lot

[ISO 3534-2]

NOTE 1 For the case of a single **specification limit** (3.13), the lot may be sentenced on the result of comparing Q with the **acceptability constant** (3.17) k.

NOTE 2 See 15.2 and 16.2.

3.19
lower quality statistic
Q_L
function of the **lower specification limit** (3.14), the sample mean, and the sample or process standard deviation

NOTE 1 For a single lower **specification limit** (3.14), the lot is sentenced on the result of comparing Q_L with the **acceptability constant** (3.17) k.

[ISO 3534-2]

NOTE 2 See Clause 4, 15.2 and 16.2.

3.20
upper quality statistic
Q_U
function of the **upper specification limit** (3.15), the sample mean, and the sample or process standard deviation

NOTE 1 For a single **upper specification limit** (3.15) the lot is sentenced on the result of comparing Q_U with the **acceptability constant** (3.17) k.

[ISO 3534-2]

NOTE 2 See Clause 4, 15.2 and 16.2.

3.21
maximum sample standard deviation
MSSD
s_{max}
largest sample standard deviation for a given sample-size code letter and **acceptance quality limit** (3.6) for which it is possible to satisfy the acceptance criterion for the combined control of double **specification limits** (3.13) when the process variability is unknown

NOTE See 15.4.

3.22
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample-size code letter and **acceptance quality limit** (3.6) for which it is possible to satisfy the acceptance criterion for the combined control of double **specification limits** (3.13) under tightened inspection when the process variability is known

NOTE See 16.3.

3.23
switching rule
instruction within an **acceptance sampling** (3.3) scheme for changing from one **acceptance sampling** (3.3) plan to another of greater or lesser severity based on demonstrated quality history

[ISO 3534-2]

NOTE 1 See Clause 20.

NOTE 2 Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of "greater or lesser severity".

3.24
measurement
Set of operations to determine the value of some quantity

[ISO 3534-2]

4 Symbols

c_{U} factor for determining the upper control limit for the sample standard deviation (see Annex H)

f_s factor that relates the maximum sample standard deviation to the difference between U and L (see Annex D)

f_σ factor that relates the maximum process standard deviation under tightened inspection to the difference between U and L (see Annex E)

k acceptability constant for the "s" method or Annex C for the "σ" method (see Annex B)

L lower specification limit (as a suffix to a variable, denotes its value at L)

μ process mean

N lot size (number of items in a lot)

n sample size (number of items in a sample)

$\hat{p}$ estimate of the process fraction nonconforming

$\hat{p}_{\mathrm{L}}$ estimate of the process fraction nonconforming below the lower specification limit

$\hat{p}_{\mathrm{U}}$ estimate of the process fraction nonconforming above the upper specification limit

p^* maximum acceptable value for the estimate of the process fraction nonconforming

P_{a} probability of acceptance

Q quality statistic

Q_{L} lower quality statistic

NOTE 1 Q_{L} is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.

Q_{U} upper quality statistic

NOTE 2 Q_{U} is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.

s sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j}^{n} (x_j - \bar{x})^2}{n-1}}$$

(See also Annex J.)

s_{max} maximum sample standard deviation (MSSD)

σ standard deviation of a process that is under statistical control

NOTE 3 σ^2 the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD)

U upper specification limit (as a suffix to a variable, denotes its value at U)

x_j measured value of the quality characteristic for the jth item of the sample

$\bar{x}$ arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n}$$

5 Acceptance quality limit (AQL)

5.1 Principle

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the acceptance quality limit may be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample-size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, contract or by the responsible authority. Where both upper and lower specification limits are given, this part of ISO 3951 addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the two limits; this is known as "combined control". (See ISO 3951-2 for "separate" and "complex" control of double specification limits.)

5.4 Preferred AQLs

The sixteen AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this part of ISO 3951 is not applicable (see 13.2).

5.5 Caution

From the above definition of the AQL, it follows that the desired protection can only be assured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

6 Switching rules for normal, tightened and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 specifies a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further specifies a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

If there is sufficient evidence from the control charts (see 19.1) that the variability is in statistical control, consideration should be given to switching to the "σ" method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see Clause 22).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses 20, 21 and 22.

7 Relation to ISO 2859-1

7.1 Similarities

a) This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans, and the preferred values used in this part of ISO 3951 are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).

c) In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample-size code letter. Then general tables give the sample size to be taken and the acceptability criterion, indexed by the sample-size code letter and the AQL. Separate tables are given for the "s" and "σ" methods, and for normal, tightened and reduced inspection.

d) The switching rules are essentially equivalent.

7.2 Differences

a) **Determination of acceptability**: Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this part of ISO 3951, two methods are considered: the "s" method for use when the process standard deviation σ is unknown, and the "σ" method for use when σ is presumed to be known. In the case of a single specification limit, the acceptability may be calculated from a formula (see 15.2 and 16.2), but for the "s" method it is also easily established by a graphical method (see 15.3). In the case of combined control of double specification limits under the "s" method, this part of ISO 3951 provides only for a graphical method of determining acceptability (see 15.4); for combined control of double specification limits under the "σ" method, a numerical method is given.

b) **Normality**: In ISO 2859 there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951 it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution or a close approximation to a normal distribution.

c) **Operating characteristic curves** (OC curves): The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves have been matched as closely as possible subject to a number of pragmatic constraints, such as keeping the sample size the same for a given code letter, and inspection method regardless of the AQL.

d) **Producer's risk**: For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1. (The producer's risks of the plans are given in Annex L.)

e) **Sample sizes**: The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters. This is particularly true for the "σ" method. (See Table A.2.)

f) **Double sampling plans**: Double sampling plans are presented separately, in ISO 3951-3.

g) **Multiple sampling plans**: No multiple sampling plans are given in this part of ISO 3951.

h) **Average Outgoing Quality Limit (AOQL)**: The AOQL concept applies when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans are generally used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

8 Limiting quality protection

8.1 Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal and reduced inspections on a continuing series of lots to provide consumer protection, while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Some users may select specific individual plans from this part of ISO 3951 and use them without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as "inspection in compliance with ISO 3951-1". When used in such a way, this part of ISO 3951 simply represents a collection of individual plans indexed by the AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

8.2 Consumer's risk quality tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give a consumer's risk quality not worse than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer's risk quality (CRQ) and a consumer's risk to be associated with it. Annex K gives values of consumer's risk quality for the "s" method and "σ" methods corresponding to a consumer's risk of 10 %.

However, application of this part of ISO 3951 to isolated lots is deprecated, as the theory of sampling by variables applies to a *process*. For isolated or short series of lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2. (See Reference [5].)

8.3 Producer's risk tables

Annex L gives the probability of non-acceptance under the "s" and "σ" methods for lots produced when the *process* fraction nonconforming equals the AQL. This probability is called the producer's risk.

8.4 Operating characteristic curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any *process* quality may, however, be judged from its operating characteristic (OC) curve. OC curves for the normal inspection "s" method sampling plans of this part of ISO 3951 are given in Charts B to R, which should be consulted when choosing a sampling plan. Also given are tables of process qualities at nine standard probabilities of acceptance for all of the "s" method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the "s" method. Most of them also provide a good approximation to the "σ" method, and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the "σ" method, refer to Annex M.

9 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge both of statistics and the product to be inspected. Clauses 10 to 13 of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable, and the choices to be made when selecting an appropriate standard plan.

10 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables, as the information obtained indicates more precisely how good the product is. Earlier warning may therefore be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted. For example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in 15.4.2 and 15.4.4.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859-1) and the standard plans in this part of ISO 3951 is given in Table A.2. It will be seen that the smallest samples are required by the "σ" method (used when the process standard deviation is presumed to be known). The sample sizes for the "s" method (used when the process standard deviation is unknown) are also in general substantially smaller than for sampling by attributes.

e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.

g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases. (For two or more quality characteristics, this part of ISO 3951 does not apply. See ISO 3951-2 for details.)

h) The use of this part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of the quality characteristic is normal. In case of doubt, the responsible authority should be consulted.

NOTE 1 ISO 5479 gives detailed procedures for tests for departure from normality.

NOTE 2 Departure from normality is also dealt with in ISO 2854:1976, Clause 2, which provides examples of graphical methods that can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

11 Choice between the "s" and "σ" methods

If it is desired to apply inspection by variables, the next question is whether to use the "s" method or the "σ" method. The "σ" method is the most economical in terms of sample size but, before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the "s" method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, whether it will be economical to change to the "σ" method. The size of the sample will generally be smaller and the acceptability criterion simpler under the "σ" method (see 16.2). On the other hand, it will still be necessary to calculate the sample standard deviation, s, for record purposes and to keep the control charts up to date. (See Clause 19.) The calculation of s can appear daunting at first sight, but the difficulty is more apparent than real; this is especially true if a calculator or computer is available. Methods of determining s and σ are given in Annex J.

12 Choice of inspection level and AQL

For a standard sampling plan, the inspection level in conjunction with the size of the lots and the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve from Charts B to R (see Clause 24) or appropriate table from Tables B to R (see Clause 24) show the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

13 Choice of sampling plan

13.1 Standard plans

The standard procedure can be used only when the production of lots is continuous.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "s" method, has been found in practice to produce workable sampling plans; but it rests on the assumption that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 20, 21 and 22), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE It is pointed out that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots is being produced), a suitable plan in this part of ISO 3951 may be selected by using Chart A (see Clause 23). Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 percent probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample-size code letter of a standard normal inspection plan, which meets the specified requirements. This should be verified by inspecting the OC curve from among Charts B to R (see Clause 24) relating to this code letter and AQL.

EXAMPLE Suppose that an acceptable value for the limiting quality is 1,5 % nonconforming and that the desired quality with a 95 % probability of acceptance is 0,15 % nonconforming. A vertical line on Chart A at 1,5 % nonconforming and a horizontal line at 0,15 % nonconforming intersect just below the sloping line indexed by the letter L. Examining Chart L, it is seen that a plan with sample-size code letter L and AQL 0,15% meets the requirements.

If the lines intersect at a point above the line marked R in Chart A, this implies that a sample of over 250 would be necessary for the "s" method and the specification cannot be met by any of the plans in this part of ISO 3951.

13.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent since, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special scheme is necessary, it should be devised only with the assistance of a statistician experienced in quality control.

14 Preliminary operations

Before starting inspection by variables,

a) check that production is considered to be continuous and that the distribution of the quality characteristic can be considered to be normal;

NOTE 1 For tests for departure from normality, see ISO 5479.

NOTE 2 If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

b) check whether the "s" method is to be used initially or whether the standard deviation is stable and known, in which case the "σ" method should be used;

c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) check, for a quality characteristic with double specification limits, that nonconformities beyond each limit are of equal importance. If this is not the case, refer to ISO 3951-2;

e) check that an AQL has been designated, and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

15 Standard procedure for the "s" method

15.1 Obtaining a plan, sampling and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

a) With the inspection level given (normally this will be II) and with the lot size, obtain the sample-size code letter using Table A.1.

b) For a single specification limit, enter Table B.1, B.2 or B.3 as appropriate with this code letter and the AQL, and obtain the sample size n and the acceptability constant k. For combined control of double specification limits when the sample size is 5 or more, find the appropriate acceptance curve from among Charts s-D to s-R.

c) Take a random sample of size n, measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean and s, the sample standard deviation (see Annex J). If $\bar{x}$ is outside the specification limit(s), the lot can be judged unacceptable without even calculating s. It is, however, necessary to calculate s for record purposes.

15.2 Acceptability criteria for single specification limits

If single specification limits are given, calculate the quality statistic

$$Q_{\mathrm{U}} = \frac{U - \bar{x}}{s}$$

or

$$Q_{\mathrm{L}} = \frac{\bar{x} - L}{s}$$

as appropriate, then compare the quality statistic (Q_{U} or Q_{L}) with the acceptability constant k obtained from either Table B.1, B.2 or B.3 for normal, tightened or reduced inspection respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if less, then it is not acceptable.

Thus, if only the upper specification limit U is given, the lot is

acceptable if $Q_{\mathrm{U}} \geqslant k$,

not acceptable if $Q_{\mathrm{U}} \leqslant k$,

or, if only the lower specification limit L is given, the lot is

acceptable if $Q_{\mathrm{L}} \geqslant k$,

not acceptable if $Q_{\mathrm{L}} \leqslant k$.

EXAMPLE 1 **Single, upper specification limit**

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From Table A.1, the sample-size code letter is F; from Table B.1 it is seen that a sample size of 13 is required and that the acceptability constant k is 1,405. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C; compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	13
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	54,615 °C
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n}(x_j - \bar{x})^2/(n-1)}$	3,330 °C
(See J.1.2, Annex J.)	
Specification limit (upper): U	60 °C

Upper quality statistic: $Q_U = (U - \bar{x})/s$	1,617
Acceptability constant: k (see Table B.1)	1,405
Acceptability criterion: Is $Q_U \geqslant k$?	Yes (1,617 > 1,405)

The lot meets the acceptability criterion, and is therefore acceptable.

EXAMPLE 2 **Single, lower specification limit**, requiring the use of an arrow in the master table.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From Table A.1 it is seen that the sample-size code letter is J, and from Table A.2 it is seen that the sample size is 35 for the "s" method. However, on entering Table B.1 with sample-size code letter J and AQL 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample-size code letter K, i.e. sample size 50 and acceptability constant k = 2,569. A random sample of size 50 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,86	6,57	6,91
6,40	6,44	6,34	6,04	6,15	6,29	6,63	6,70	6,67	6,67
6,44	7,15	6,70	6,59	6,51	6,80	5,94	5,92	6,56	6,53
6,35	7,17	6,83	6,25	6,96	7,00	6,38	6,83	6,29	6,39
6,80	5,84	6,16	6,25	6,57	6,71	6,77	6,55	6,87	6,25

Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	50
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	6,542 s
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n}(x_j - \bar{x})^2/(n-1)}$	0,3120 s
(See J.1.2, Annex J.)	
Lower specification limit: L	4,0 s
Lower quality statistic: $Q_L = (\bar{x} - L)/s$	8,147
Acceptability constant: k (see Table B.1)	2,569
Acceptability criterion: Is $Q_L \geqslant k$?	Yes (8,147 > 2,569)

The lot meets the acceptability criteria, so it is acceptable.

15.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$\bar{x} = U - ks$ (for an upper limit)

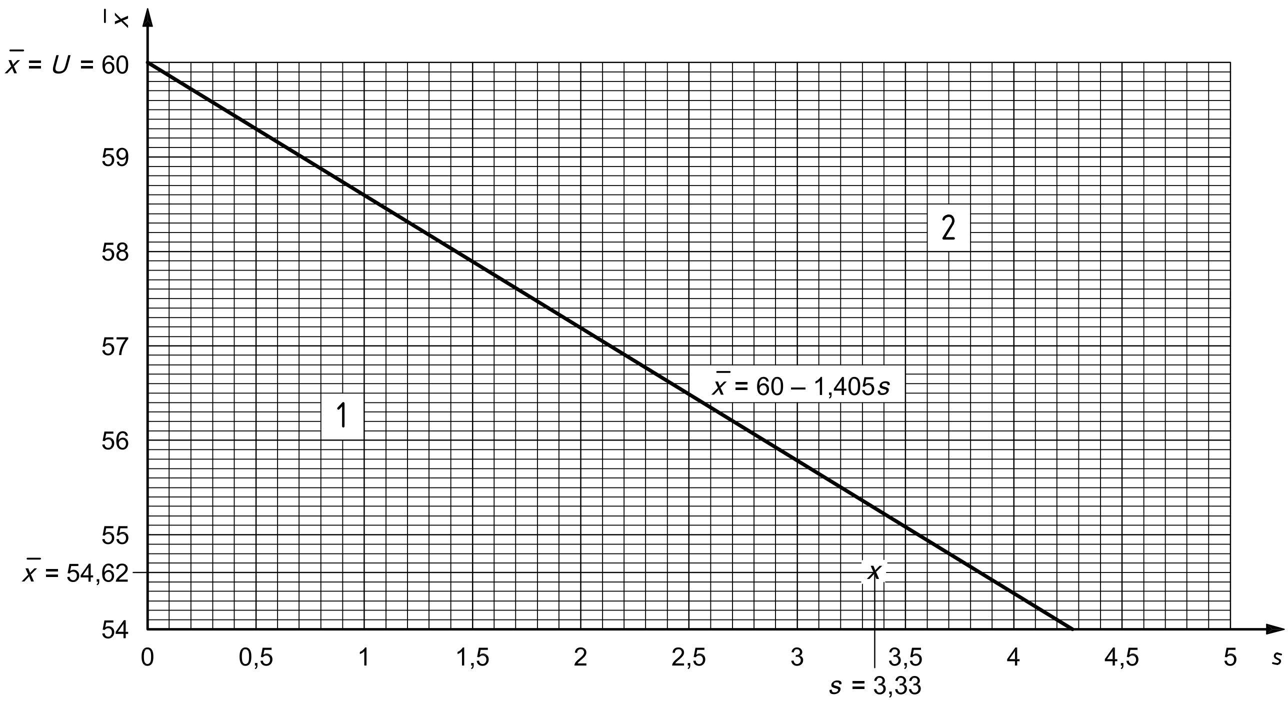
or

$\bar{x} = L + ks$ (for a lower limit),

as appropriate, on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit, the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Plot the point (s, $\bar{x}$) on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE Using the data given in Example 1 of 15.2, mark the point U = 60 on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $-k$. [As k = 1,405, this means that the line passes through the points (s = 1, $\bar{x}$ = 58,595), (s = 2, $\bar{x}$ = 57,190), (s = 3, $\bar{x}$ = 55,785), etc.]. Select a suitable point and draw a straight line through it and (s = 0, $\bar{x}$ = 60), i.e. (0, U). The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3,330 and 54,615. The point (s, $\bar{x}$) is seen from Figure 1 to lie within the accept zone; the lot is therefore acceptable.

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot, the point (s, $\bar{x}$) may be plotted to determine whether or not the lot is acceptable.



Key

1 accept zone

2 reject zone

Figure 1 — Example of the use of an acceptance chart for single specification limit "s" method

15.4 Acceptability criterion for combined control of double specification limits

15.4.1 General

For the "s" method with combined control of the upper and lower specification limits, i.e. with an overall AQL for the percentage of items from the process outside the specification limits, this part of ISO 3951 provides a graphical method for determining lot acceptability for all sample sizes except for sample sizes 3 and 4. (ISO 3951-2 provides numerical methods only.) The larger the sample variability, the less likely it is that the requirement is satisfied. If the value of s exceeds the value of the maximum sample standard deviation

(MSSD) obtained from Table D.1, D.2 or D.3, no further calculation or reference to graphs is required, since the lot shall be immediately judged unacceptable.

Numerical methods are provided for the combined control of double specification limits for sample sizes 3 and 4.

15.4.2 Procedure for sample size 3

It may be seen from Annex B that the required sample size is 3 for the "s" method for sample-size code letter B under normal and tightened inspection and for sample-size code letters B to D under reduced inspection.

After calculating the sample mean $\bar{x}$ and the sample standard deviation s, find the applicable value of the coefficient f_s from the first row of Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$s_{max} = (U - L)f_s$$

Then compare s with s_{max}. If s is greater than s_{max} then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. Multiply Q_U and Q_L by $\sqrt{3}/2$ (i.e. approximately 0,866) and use Table F.1 to determine the estimates $\hat{p}_U$ and $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and lower limits respectively.

NOTE 1 Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0,5 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this part of ISO 3951. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming can be obtained by entering Table F.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. For example if $Q_U = -0{,}156$ then $\sqrt{3}Q_U/2 = -0{,}135$; entering Table F.1 with 0,135 gives an estimate of 0,4569; subtracting this from 1,0 gives $\hat{p}_U = 0{,}5431$.

NOTE 2 The basis of Table F.1 is given in Annex K. Instead of using Table F.1, the estimate of the process fraction nonconforming beyond each specification limit when $n = 3$ may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3}, \\ \dfrac{2}{\pi}\arcsin\left\{\sqrt{(1 - Q\sqrt{3}/2)/2}\right\} & \text{if } -2/\sqrt{3} \leqslant Q \leqslant 2/\sqrt{3}, \\ 1 & \text{if } Q < -2/\sqrt{3}. \end{cases}$$

These two estimates must be added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table G.1, the lot is considered to be acceptable; otherwise, the lot is considered to be not acceptable.

EXAMPLE Determination of acceptability for combined control of double specification limits when the sample size is 3.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so combined control of the double specification limits is appropriate. The specification limits are set at 10 m either side of the target point at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table A.1, the sample-size code letter is found to be B. From Table A.2, it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding errors of –5,0 m, 6,7 m and 8,8 m. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	Value obtained
Sample size: n	3
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	3,5 m
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n}(x_j - \bar{x})^2/(n-1)}$	7,436 m
(See J.1.2, Annex J.)	
Value of f_s for MSSD (s_{max}) (Table D.1)	0,474
$s_{max} = (U - L)f_s = [10 - (-10)] \times 0,474$	9,48

As $s = 7,436 < s_{max} = 9,48$, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x})/s = (10 - 3,5)/7,436$	0,8741
$Q_L = (\bar{x} - L)/s = (3,5 + 10)/7,436$	1,815
$\sqrt{3}Q_U/2$	0,757
$\sqrt{3}Q_L/2$	1,572
$\hat{p}_U$ (from Table F.1)	0,2267
$\hat{p}_L$ (from Table F.1)	0,0000
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,2267
p^* (from Table G.1, normal inspection)	0,1905

As $\hat{p} > p^*$, the lot is not acceptable.

NOTE 3 This lot is not acceptable even though all inspected items *in the sample* are within the specification limits.

15.4.3 Procedure for sample size 4

It may be seen from Annex B that the required sample size is 4 for the "s" method for sample-size code letter C under normal and tightened inspection and for sample-size code letter E under reduced inspection.

After calculating the sample mean $\bar{x}$ and the sample standard deviation s, find the applicable value of the coefficient f_s from the second row of Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$s_{max} = (U - L)f_s$$

Then compare s with s_{max}. If s is greater than s_{max} then the lot may be rejected without further calculation.

Otherwise determine the values of $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. Calculate

$$\hat{p}_U = \begin{cases} 0 & \text{if } Q_U > 1,5, \\ 0,5 - Q_U/3 & \text{if } -1,5 \leqslant Q_U \leqslant 1,5, \\ 1 & \text{if } Q_U < -1,5, \end{cases} \tag{1}$$

and

$$\hat{p}_L = \begin{cases} 0 & \text{if } Q_L > 1{,}5, \\ 0{,}5 - Q_L/3 & \text{if } -1{,}5 \leqslant Q_L \leqslant 1{,}5, \\ 1 & \text{if } Q_L < -1{,}5. \end{cases} \tag{2}$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table G.1, the lot is considered to be acceptable; otherwise the lot is considered to be not acceptable.

NOTE The basis of Equations (1) and (2) is given in Annex N.

EXAMPLE Items are being manufactured in lots of size 50. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming using an AQL of 2,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table A.1, the sample-size code letter is found to be C. From Table A.2, it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters 82,4 mm, 82,2 mm, 83,1 mm and 82,3 mm. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Value obtained**
Sample size: n	4
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	82,50 mm
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n}(x_j - \bar{x})^2/(n-1)}$	0,4082 mm
(See J.1.2, Annex J.)	
Upper specification limit: U	84,0 mm
Lower specification limit: L	82,0 mm
Value of f_s for MSSD (s_{max}) (Table D.1)	0,376
$s_{max} = (U - L)f_s = (84 - 82) \times 0{,}376$	0,752 mm

As $s = 0{,}408\,2 < s_{max} = 0{,}752$, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x})/s = (84 - 82{,}5)/0{,}408\,2$	3,674 7
$Q_L = (\bar{x} - L)/s = (82{,}5 - 82)/0{,}408\,2$	1,224 9
$\hat{p}_U$ [from (1) above]	0,000 0
$\hat{p}_L$ [from (2) above]	0,091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,091 7
p^* (from Table G.1, normal inspection)	0,112 3

As $\hat{p} < p^*$, the lot is acceptable.

15.4.4 Procedure for sample sizes greater than 4

After calculating the sample mean $\overline{x}$ and the sample standard deviation s, find the applicable value of the coefficient f_s from Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$s_{max} = (U - L)f_s$$

Then compare s with s_{max}. If s is greater than s_{max} then the lot may be rejected without further ado.

Otherwise, from among Charts s-D to s-R, consult the chart labelled with the appropriate sample-size code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of $s/(U - L)$ and $(\overline{x} - L)/(U - L)$ and plot a point representing these values on a copy of the graph. If the point lies inside the curve, the lot is acceptable; if outside, the lot is not acceptable.

For greater convenience it is recommended that, before the inspection operations begin, the required acceptance curves for normal, tightened and reduced inspection be copied. The scales should be adjusted so that s and $\overline{x}$ can be plotted directly (i.e. the upper specification limit is given instead of 1,0 and the lower specification limit instead of 0,0 on the vertical scale).

Then plot a point on the chart representing the values of s and $\overline{x}$ found from the sample. If the point lies inside or on the curve, the lot is acceptable; if outside, it is not acceptable.

EXAMPLE The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From Table A.1, the sample-size code letter is found to be F; from Table A.2 it is seen that a sample of 13 is required, and from Table D.1 that the value of f_s for the MSSD under normal inspection is 0,274. Suppose the measurements obtained are as follows:

65,5 °C; 60,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C;

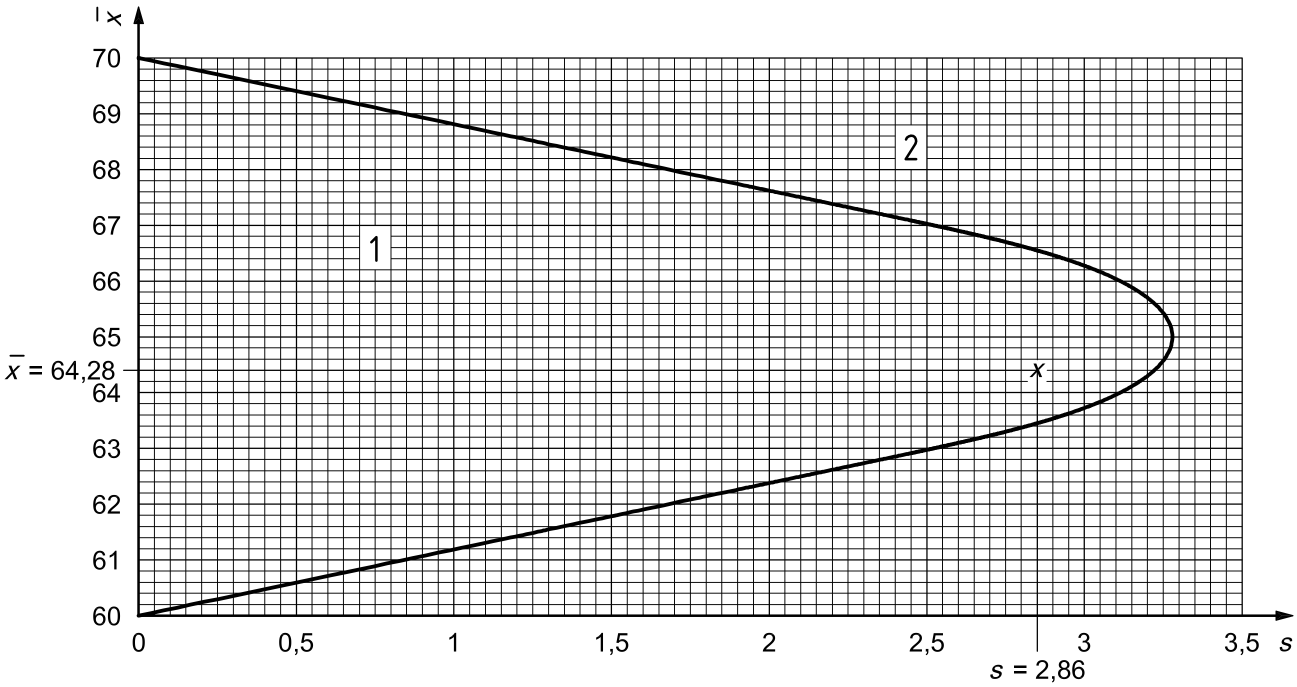
compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Sample size: n	13
Sample mean: $\overline{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	64,28 °C
Sample standard deviation: $s = \sqrt{\sum_{j=1}^{n}(x_j - \overline{x})^2/(n-1)}$	2,86 °C
(See J.1.2, Annex J.)	
Upper specification limit: U	70,0 °C
Lower specification limit: L	60,0 °C
Value of f_s for MSSD (s_{max}) (Table D.1)	0,274
$s_{max} = (U - \mathrm{L})f_s$	2,74 °C

As the value of s exceeds s_{max} the lot may immediately be judged unacceptable.

NOTE This lot is not acceptable even though all inspected items *in the sample* are within the specification limits.

Suppose that the AQL had been 4,0 % instead of 1,5 %. In that case the value of f_S would be 0,328, giving an MSSD of 3,28. As s is now less than the MSSD, it is not possible to determine at this stage whether or not the lot is acceptable. The appropriate acceptance curve is taken from Chart s-F. If, as on Figure 2, the scales have been adjusted to the real measurements, plot the point (s = 2,86; $\bar{x}$ = 64,28). This lies inside the acceptance curve for an AQL of 4,0 %, so the lot is acceptable.



Key

1 accept zone

2 reject zone

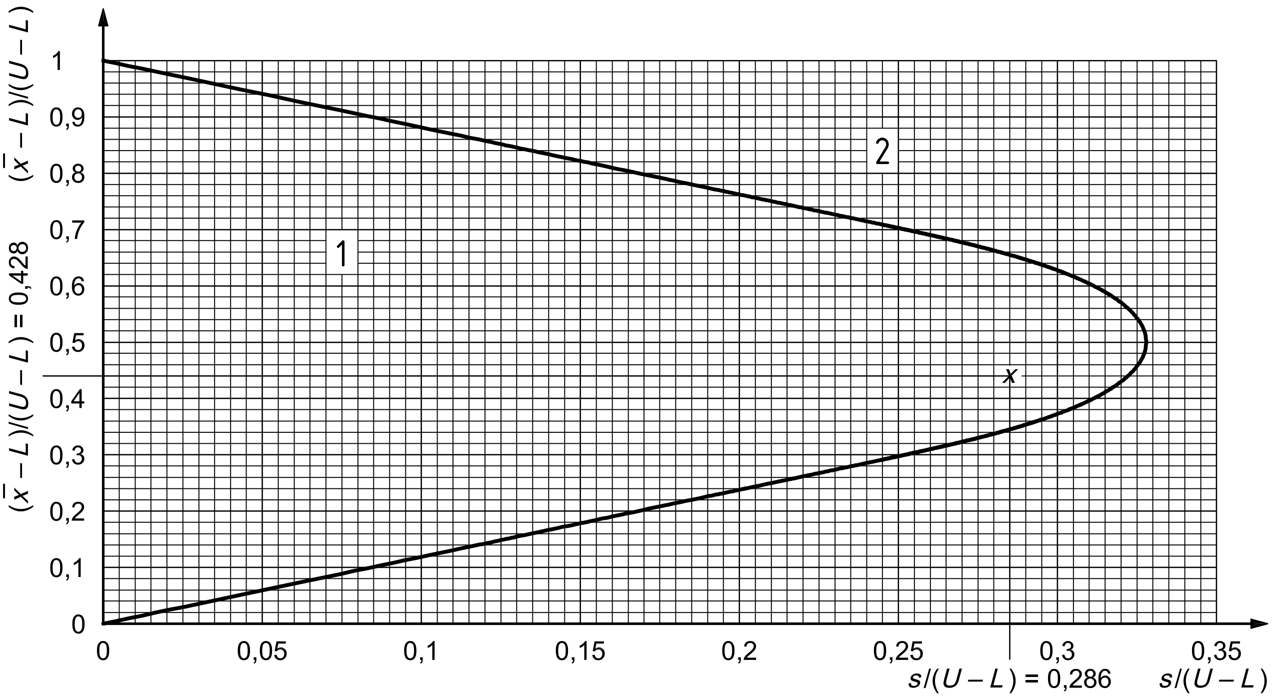
Figure 2 — Example of the use of an acceptance chart for combined control of double specification limit "s" method with actual scales

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required:

standardized sample mean: $(\bar{x} - L)/(U - L)$ = (64,28 – 60)/(70 – 60) = 0,428.

standardized sample standard deviation: $s/(U - L)$ = 2,86/(70 – 60) = 0,286.

The point (0,286, 0,428) is plotted on Figure 3. As it lies inside the acceptance curve for AQL = 4,0 %, the lot is acceptable.



Key

1 accept zone

2 reject zone

Figure 3 — Example of the use of an acceptance chart for combined control of double specification limit "*s*" method with normalized scales

16 Standard procedure for the "σ" method

16.1 Obtaining a plan, sampling and preliminary calculations

The "σ" method is only to be used when there is valid evidence that the standard deviation, σ, of the process can be considered constant with a known value.

From Table A.1 obtain the sample-size code letter. Then, depending on the severity of inspection, enter Table C.1, C.2 or C.3 with the sample-size code letter and the specified AQL to obtain the sample size n and acceptability constant k.

Take a random sample of this size, measure the characteristic under inspection, x, for all items of the sample and calculate the mean $\bar{x}$. (The sample standard deviation s should also be calculated, but only for the purpose of checking the continued stability of the process standard deviation. See Clause 19.)

16.2 Acceptability criteria for a single specification limit

The acceptability criterion can be found by following the procedure given for the "s" method. First replace the s derived from the individual samples by σ, the presumed known value of the standard deviation of the process, and then compare the calculated value of Q with the value of the acceptability constant k obtained from one of the Tables C.1, C.2 and C.3.

Note, for example, that the acceptability criterion $Q_U \; [= (U - \bar{x})/\sigma] \geqslant k$ for an upper specification may be written as $\bar{x} \leqslant U - k\sigma$. As U, k and σ are all known in advance, the acceptance value $\bar{x}_U \; [= U - k\sigma]$ should therefore be determined before inspection begins. For an upper specification limit, a lot will be

acceptable if $\bar{x} \leqslant \bar{x}_U$ [= $U - k\sigma$]; not acceptable if $\bar{x} > \bar{x}_U$ [= $U - k\sigma$].

Similarly, for a lower specification limit, a lot will be

acceptable if $\bar{x} \geqslant \bar{x}_L$ [$= L + k\sigma$]; not acceptable if $\bar{x} < \bar{x}_L$ [$= L + k\sigma$].

EXAMPLE The specified minimum yield point for certain steel castings is 400 N/mm^2. A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. The value of σ is considered to be 21 N/mm^2. From Table A.1 it is seen that the sample-size code letter is H. Then, from Table C.1, it is seen that for an AQL of 1,5 % the sample size n is 12 and the acceptability constant k is 1,613. Assume that the yield points of the sample specimens are 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400; 445. Compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Acceptability constant: k	1,613
Known: σ	21 N/mm^2
Product: $k\sigma$	33,9 N/mm^2
Specification limit: L	400 N/mm^2
Acceptance value: $\bar{x}_L = L + k\sigma$	433,9 N/mm^2
Sum of measurement results: $\sum_{j=1}^{n} x_j$	5 184 N/mm^2
Sample size: n	12
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	429,8 N/mm^2
Acceptability criterion: is $\bar{x} \geqslant \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion, so the lot is not acceptable.

16.3 Acceptability criterion for combined control of double specification limits

For combined control of the upper and the lower specification limits, i.e. where there is an overall AQL for the percentage of the process outside the specification limits, the following procedure is recommended.

a) Before sampling, enter Table E.1 with the AQL to determine the value of the factor f_σ

b) Calculate the maximum allowable value of the process standard deviation, using the formula $\sigma_{max} = (U - L)f_\sigma$ for the MPSD.

c) Compare the value of the process standard deviation σ with σ_{max}. If σ exceeds σ_{max} the process is unacceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

d) If $\sigma \leqslant \sigma_{max}$ then use the lot size and given inspection level to determine the sample-size code letter from Table A.1.

e) From the sample-size code letter and inspection severity (i.e. whether inspection is normal, tightened or reduced) determine the sample size, n, and acceptability constant, k, from Table C.1, C.2 or C.3 as appropriate.

f) Calculate the upper allowable bound, $\bar{x}_U$, to sample means from the formula $\bar{x}_U = U - k\sigma$, and the lower allowable bound $\bar{x}_L$ from the formula $\bar{x}_L = L + k\sigma$.

g) Select a random sample of size n from the lot and calculate the sample mean $\bar{x}$. The acceptability criterion is: If $\bar{x}_L \leqslant \bar{x} \leqslant \bar{x}_U$, the lot is acceptable; if $\bar{x} < \bar{x}_L$ or $\bar{x} > \bar{x}_U$, the lot is not acceptable.

EXAMPLE The specification for electrical resistance of a certain electrical component is (520 ± 50) Ω. Production is at a rate of 2 500 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 4 %, is to be used for the two specification limits (470 and 570). The value of σ is known to be 21,0. Entering Table A.1 with the lot size and inspection level, it is found that the sample-size code letter is K; from Table A.2 it is seen that a sample size of 18 is required under normal inspection. Suppose the values of the sample resistance, in ohms, are as follows: 515; 491; 479; 507; 543; 521; 536; 483; 509; 514; 507; 484; 526; 552; 499; 530; 512; 492.

Information needed	**Value obtained**
Factor from Table E.1: f_σ	0,223
Upper specification limit: U	570 Ω
Lower specification limit: L	470 Ω
Maximum process standard deviation, $\sigma_{max} = (U - L) f_\sigma$	22,3 Ω
Known: σ	21,0 Ω

(As σ is less than σ_{max}, the sample is analysed further for lot acceptability.)

Sample size: n	18
Acceptability constant (from Table C.1)	1,340
Upper bound for $\bar{x}$: $\bar{x}_U = U - k\sigma$	541,9 Ω
Lower bound for $\bar{x}$: $\bar{x}_L = L + k\sigma$	498,1 Ω
Sum of measurement results: $\sum_{j=1}^{n} x_j$	9 200 Ω
Sample mean: $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$	511,1 Ω

As $\bar{x}$ at 511,1 Ω lies between the acceptance limits for $\bar{x}$ of 498,1 Ω and 541,9 Ω, the lot is acceptable.

All the calculations other than the last two lines should be completed before sampling begins.

If, for example, σ had been known to be 25, then σ exceeds the MPSD and therefore sampling inspection should not even have taken place.

17 Procedure during continuing inspection

As a variables sampling inspection plan can only operate efficiently if

a) the characteristic being inspected is normally distributed,

b) records are kept,

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

18 Normality and outliers

18.1 Normality

The responsible authority should check for normality before sampling begins. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables, or whether use should be made of the tests for departure from normality given in ISO 5479 or in ISO 2854:1976, Clause 2.

18.2 Outliers

An outlier (or an outlying observation) is a datum that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, will produce an increase in variability and change the mean and may consequently lead to non-acceptance of the lot (see, for example, ISO 5725-2). When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and purchaser.

19 Records

19.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and a warning given before an unacceptable standard is reached, but this is only possible if adequate records are kept.

Whatever the method used, "s" or "σ", records should be kept of the values of $\bar{x}$ and s, preferably in the form of control charts (see ISO 7870 and ISO 8258).

This procedure should be applied especially with the "σ" method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ.

For combined control of double specification limits, the values of the MSSD, given in Table D.1, D.2 or D.3, should be plotted on the s control chart, as an indication of an *unacceptable* value.

NOTE Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedures given in Clauses 15 and 16.

19.2 Lots that are not accepted

Particular care shall be taken to record all lots that are not accepted and to see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

20 Operation of switching rules

The standard switching rules are as follows.

a) **Normal inspection** is used at the start of inspection (unless otherwise designated) and shall continue to be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

b) **Tightened inspection** shall be instituted when two lots on original normal inspection are not accepted within any five or fewer successive lots.

Tightened inspection is generally achieved by increasing the value of the acceptability constant k. The values are tabulated in Table B.2 for the "s" method and Table C.2 for the "σ" method. For neither method is there a change in the size of the sample in switching from normal to tightened inspection, unless the AQL is so small that the tables indicate, with a downward arrow, that an increase in sample size is necessary.

c) **Tightened inspection** shall be **relaxed** when five successive lots on original inspection have been accepted on tightened inspection; then normal inspection shall be reinstated.

d) **Reduced inspection** may be instituted after ten successive lots have been accepted under normal inspection, provided that

1) these lots would have been acceptable if the AQL had been one step tighter;

NOTE If a value of k for this tighter AQL is not given in Table B.1 ("s" method) or Table C.1 ("σ" method), refer to Table I.1.

2) production is in statistical control;

3) reduced inspection is considered desirable by the responsible authority.

Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of n and k for reduced inspection are given in Table B.3 for the "s" method and Table C.3 for the "σ" method.

e) **Reduced inspection** shall **cease** and normal inspection be reinstated if any of the following occur on original inspection:

1) a lot is not accepted;

2) production becomes irregular or delayed;

3) reduced inspection is no longer considered desirable by the responsible authority.

21 Discontinuation and resumption of inspection

If the cumulative number of lots not accepted in a sequence of consecutive lots on original tightened inspection reaches 5, the acceptance procedures of this part of ISO 3951 shall be discontinued.

Inspection under the provisions of this part of ISO 3951 shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service, and the responsible authority has agreed that this action is likely to be effective. Tightened inspection shall then be used as if 20 b) had been invoked.

22 Switching between the "s" and "σ" methods

22.1 Estimating the process standard deviation

While this part of ISO 3951 is being used, the weighted root mean square of the values of s shall be calculated periodically as estimates of the process standard deviation σ for both the "s" and the "σ" methods. (See J.2 in Annex J.) The value of σ shall be re-estimated at five lot intervals, unless the responsible authority specifies another interval. The estimate shall be based on the preceding 10 lots, unless the responsible authority specifies another number of lots.

22.2 State of statistical control

Calculate the upper control limit for each of the 10 lots (or other number of lots specified by the responsible authority) from the expression $c_u\sigma$, where c_u is a factor that depends on the sample size n and is given in Table H.1. If none of the sample standard deviations, s_j, exceeds the corresponding control limit, then the process may be considered to be in a state of statistical control; otherwise the process shall be considered to be out of statistical control.

NOTE 1 If the sample sizes from the lots are all equal, then the value of $c_u\sigma$ is common to all the lots.

NOTE 2 If the sample sizes from each lot vary, it is not necessary to calculate $c_u\sigma$ for those lots for which the sample standard deviation, s_j, is less than or equal to σ.

22.3 Switching from the "s" method to the "σ" method

If the process is considered to be in a state of statistical control under the "s" method, then the "σ" method may be instituted using the latest value of σ.

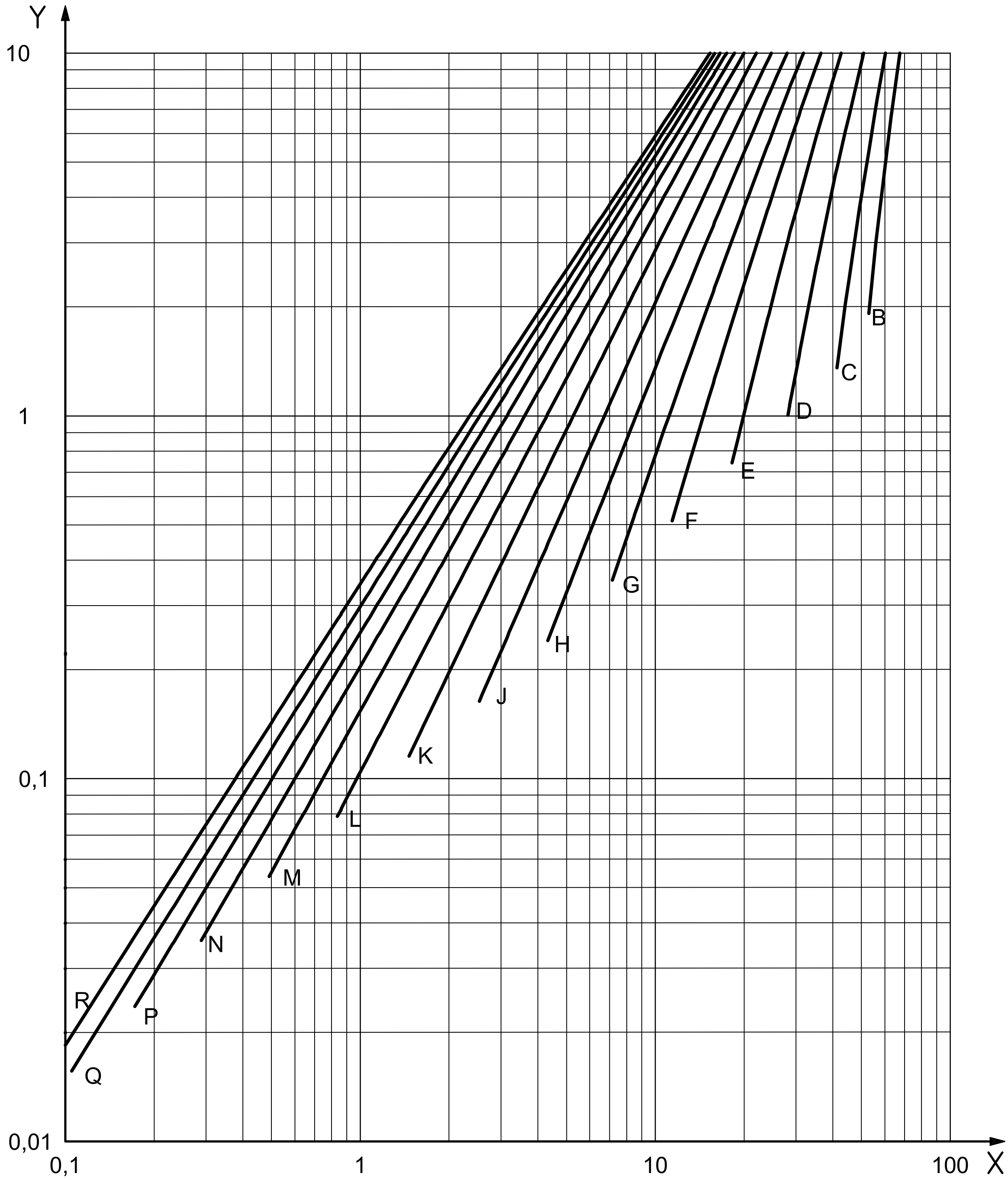
NOTE This switch is made at the discretion of the responsible authority.

22.4 Switching from the "σ" method to the "s" method

It is recommended that a control chart for s be kept, even under the "σ" method. Once the process is considered to be out of statistical control, inspection shall be switched to the "s" method.

23 Chart A — Sample-size code letters of standard single sampling plans for specified quality levels

Figure 4 shows sample-size code letters of standard single sampling plans for specified quality levels at 95 % and 10 % probabilities of acceptance (in percent nonconforming).



Key

X limiting quality, i.e. quality level at 10 % probability of acceptance (in percent nonconforming)

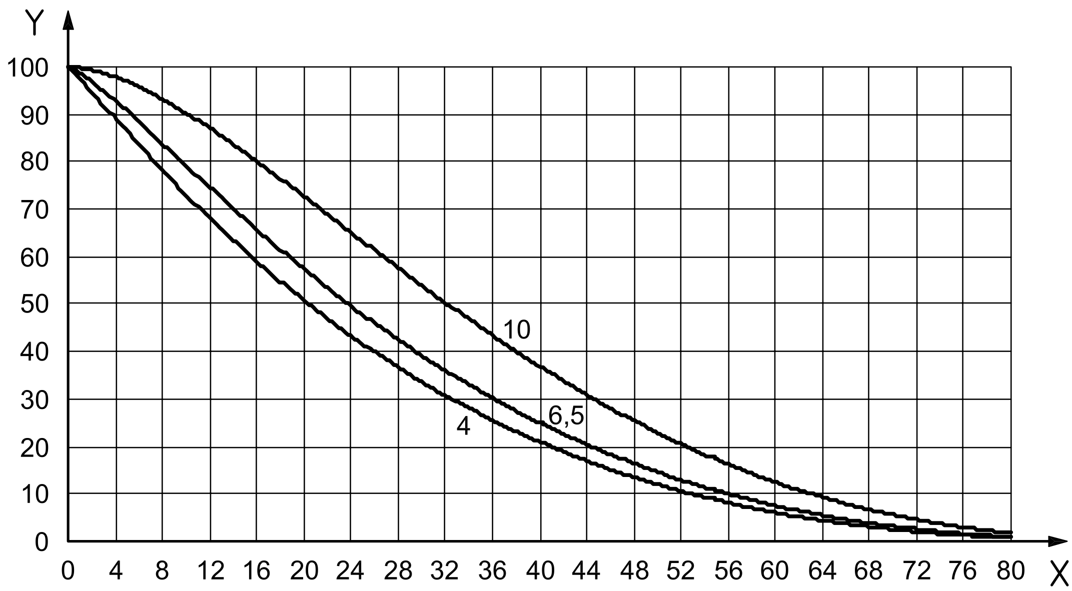
Y quality level at 95 % probability of acceptance (in percent nonconforming)

Sample-size code letters are shown in boldface type.

Figure 4 — Chart A — Sample-size code letters of standard single sampling plans for specified quality levels at 95 % and 10 % probabilities of acceptance

24 Charts B to R (Figures 5 to 19) — Operating characteristic curves and tabulated values for sample-size code letter B to R: "*s*" method

24.1 Chart B



Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

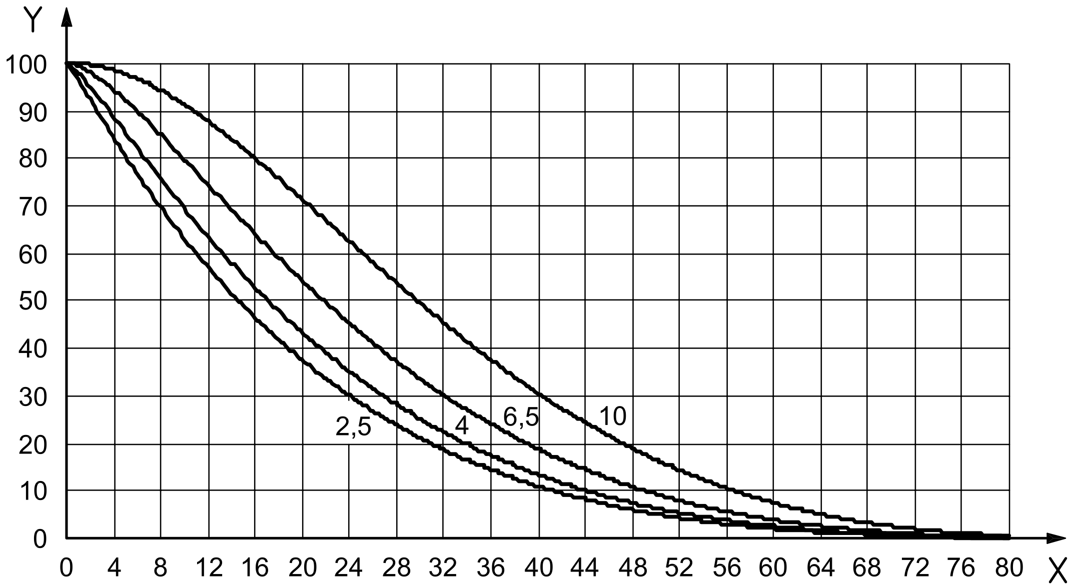
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

P_a	**Acceptance quality limit (normal inspection) — Sample-size code letter B**					P_a
	4,0		6,5	10,0		
99,0	0,45	0,57	0,85	2,61	8,63	99,0
95,0	1,92	2,25	2,99	6,63	16,60	95,0
90,0	3,69	4,19	5,29	10,18	22,35	90,0
75,0	9,25	10,09	11,83	18,77	34,10	75,0
50,0	20,40	21,54	23,81	32,20	49,19	50,0
25,0	36,45	37,65	40,01	48,34	64,40	25,0
10,0	52,92	53,97	56,01	63,09	76,41	10,0
5,0	62,52	63,42	65,15	71,15	82,37	5,0
1,0	77,98	78,56	79,67	83,53	90,71	1,0
	6,5		10,0			
	Acceptance quality limit (tightened inspection) — Sample-size code letter B					
	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letters B-D					

Figure 5 — Chart B — Operating characteristics curves for single sample plans, normal inspection

24.2 Chart C



Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

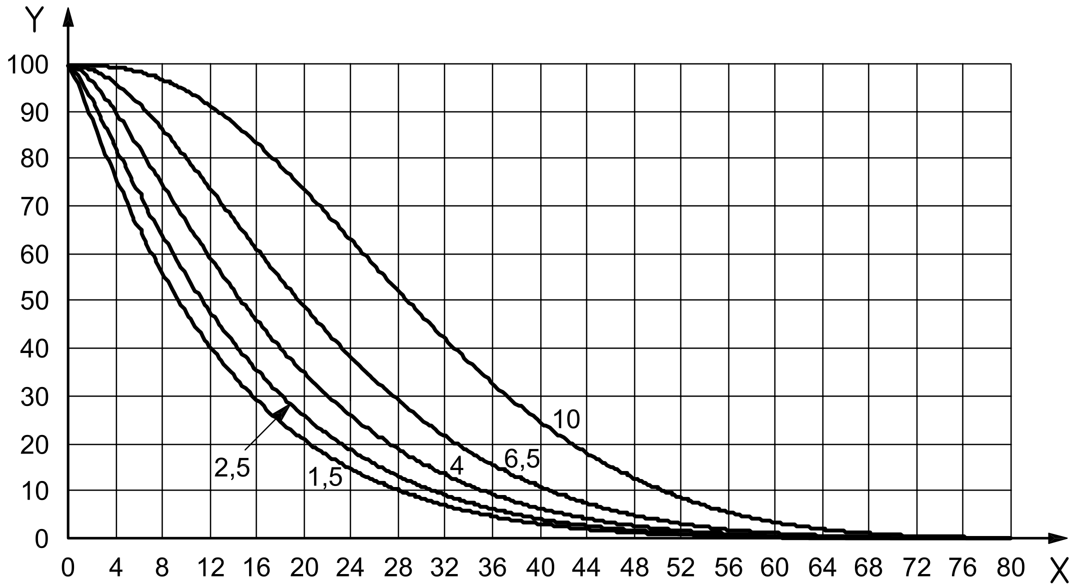
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

P_a	Acceptance quality limit (normal inspection) — Sample-size code letter C						P_a
	2,5		4,0	6,5	10,0		
99,0	0,34	0,42	0,59	1,33	3,41	10,42	99,0
95,0	1,36	1,59	2,03	3,70	7,47	17,98	95,0
90,0	2,58	2,93	3,61	5,94	10,76	23,15	90,0
75,0	6,46	7,08	8,21	11,78	18,34	33,38	75,0
50,0	14,59	15,50	17,10	21,84	29,76	46,37	50,0
25,0	27,17	28,24	30,09	35,31	43,53	59,76	25,0
10,0	41,32	42,37	44,16	49,09	56,61	70,93	10,0
5,0	50,30	51,27	52,90	57,40	64,15	76,84	5,0
1,0	66,36	67,10	68,33	71,69	76,68	85,89	1,0
	4,0		6,5	10,0			
	Acceptance quality limit (tightened inspection) — Sample-size code letter C						
	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter E						

Figure 6 — Chart C — Operating characteristics curves for single sample plans, normal inspection

24.3 Chart D



Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Tabulated values for operating characteristic curves for single sampling plans

All data in percent

P_a	**Acceptance quality limit (normal inspection) — Sample-size code letter D**							P_a
	1,5		2,5	4,0	6,5	10,0		
99,0	0,29	0,36	0,50	1,00	2,12	5,43	13,35	99,0
95,0	1,01	1,19	1,53	2,60	4,66	9,75	20,34	95,0
90,0	1,81	2,08	2,58	4,08	6,76	12,89	24,82	90,0
75,0	4,27	4,76	5,60	7,95	11,73	19,53	33,34	75,0
50,0	9,45	10,21	11,47	14,78	19,68	28,88	43,91	50,0
25,0	17,87	18,86	20,47	24,48	30,07	39,88	54,94	25,0
10,0	28,19	29,28	31,02	35,26	40,94	50,49	64,57	10,0
5,0	35,31	36,40	38,14	42,30	47,79	56,85	69,96	5,0
1,0	49,55	50,54	52,10	55,78	60,52	68,16	78,91	1,0
	2,5		4,0	6,5	10,0			
	Acceptance quality limit (tightened inspection) — Sample-size code letter D							
	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter F							

Figure 7 — Chart D — Operating characteristics curves for single sample plans, normal inspection

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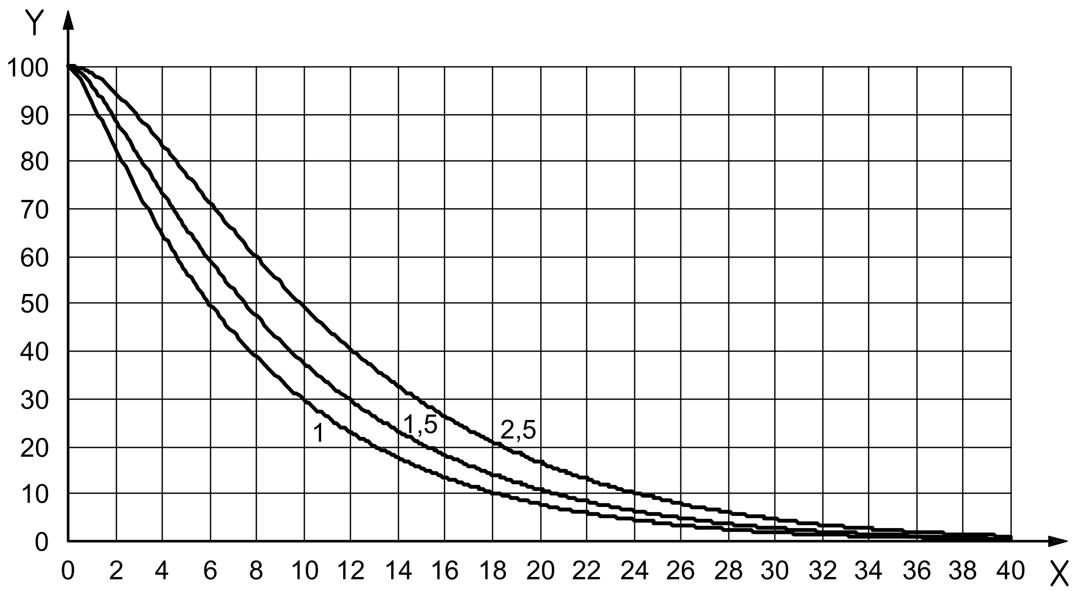
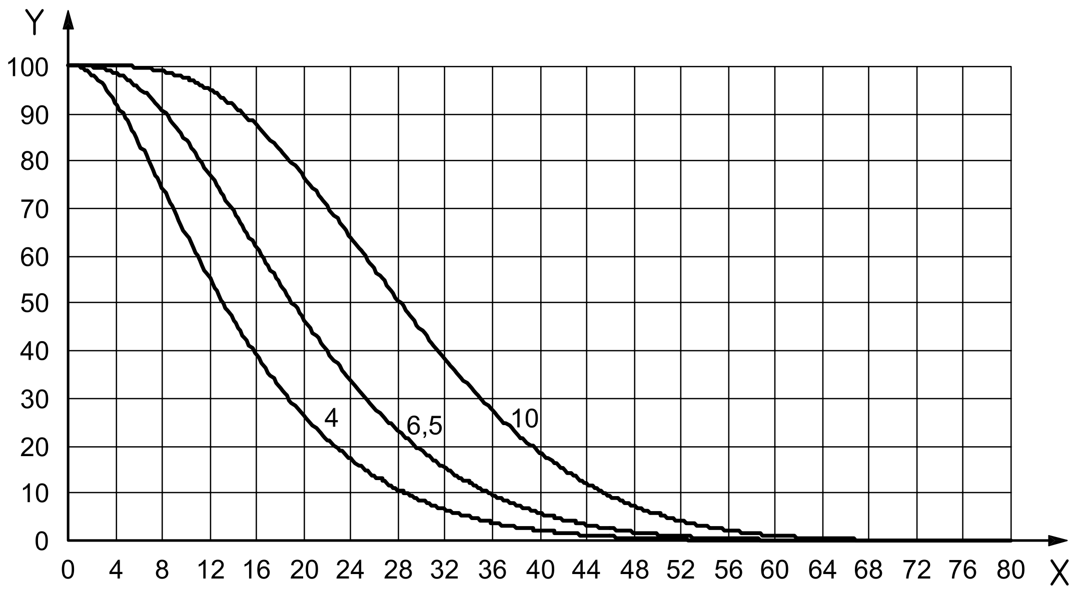
24.4 Chart E



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 8 — Chart E — Operating characteristics curves for single sample plans, normal inspection

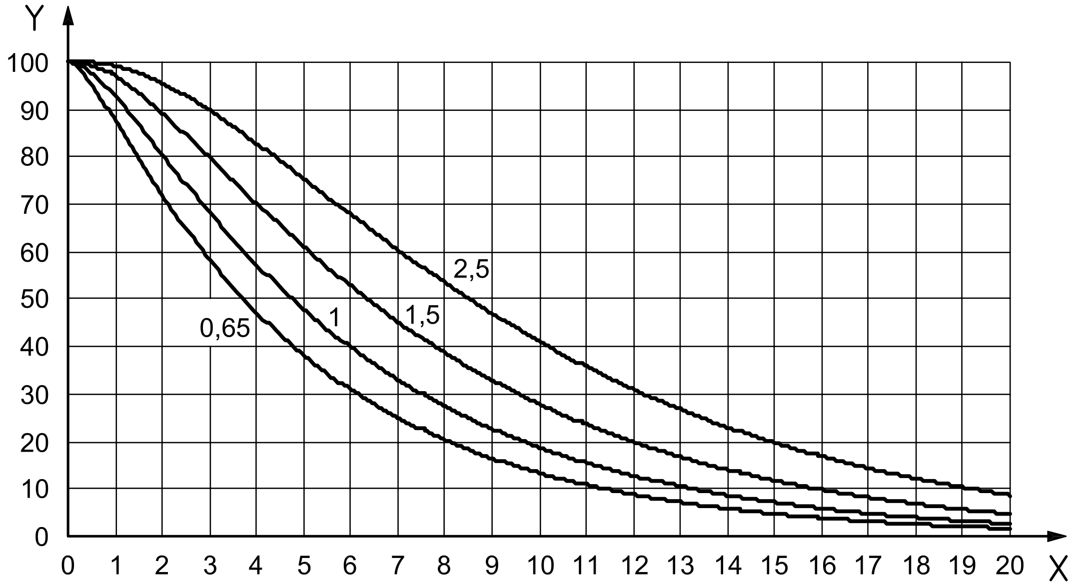
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

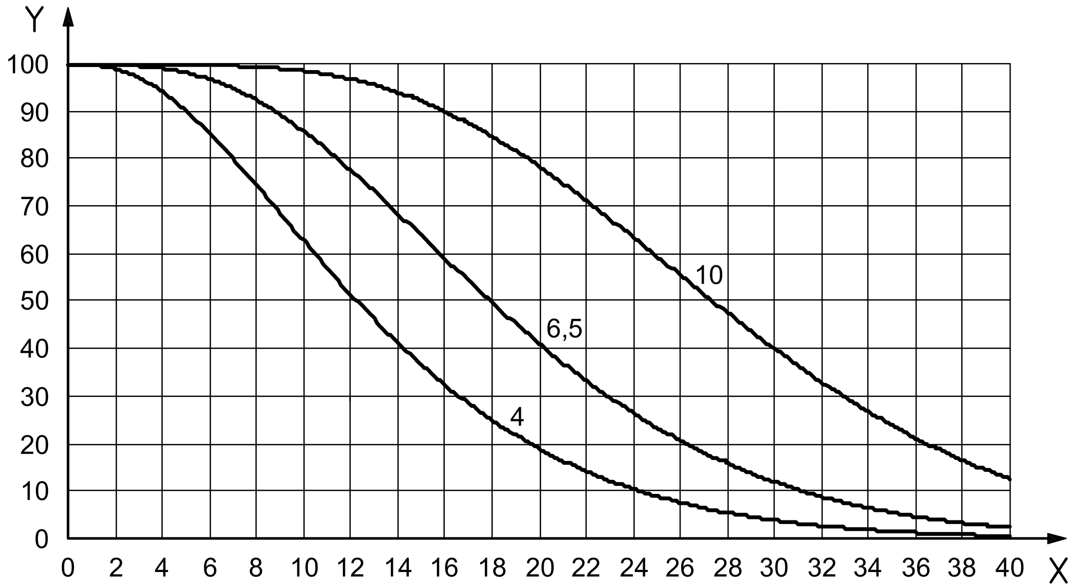
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter E								P_a
	1,0		1,5	2,5	4,0	6,5	10,0		
99,0	0,24	0,30	0,41	0,78	1,51	3,40	7,63	12,27	99,0
95,0	0,74	0,88	1,13	1,88	3,19	6,13	11,91	17,74	95,0
90,0	1,26	1,47	1,83	2,86	4,56	8,15	14,78	21,20	90,0
75,0	2,81	3,17	3,77	5,38	7,83	12,55	20,50	27,78	75,0
50,0	6,00	6,58	7,51	9,86	13,18	19,05	28,18	36,08	50,0
25,0	11,32	12,13	13,39	16,43	20,48	27,19	36,98	45,08	25,0
10,0	18,20	19,17	20,66	24,15	28,61	35,67	45,52	53,42	10,0
5,0	23,24	24,27	25,84	29,48	34,03	41,10	50,74	58,35	5,0
1,0	34,16	35,23	36,84	40,48	44,92	51,59	60,41	67,20	1,0
	1,5		2,5	4,0	6,5	10,0			
	Acceptance quality limit (tightened inspection) — Sample-size code letter E								
	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter G								

Figure 8 *(continued)*

24.5 Chart F



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 9 — Chart F — Operating characteristics curves for single sample plans, normal inspection

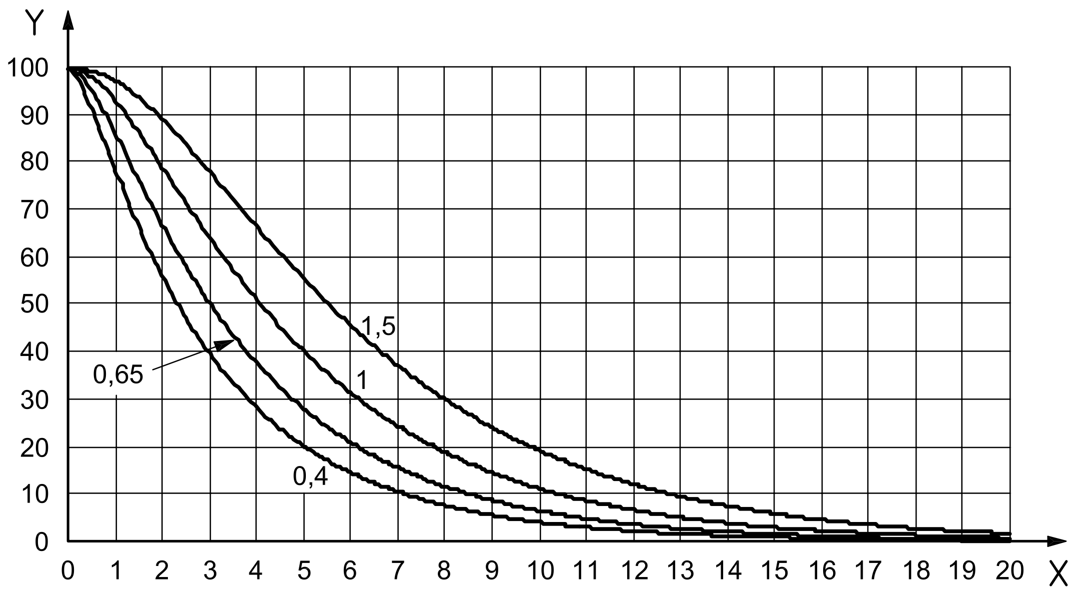
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

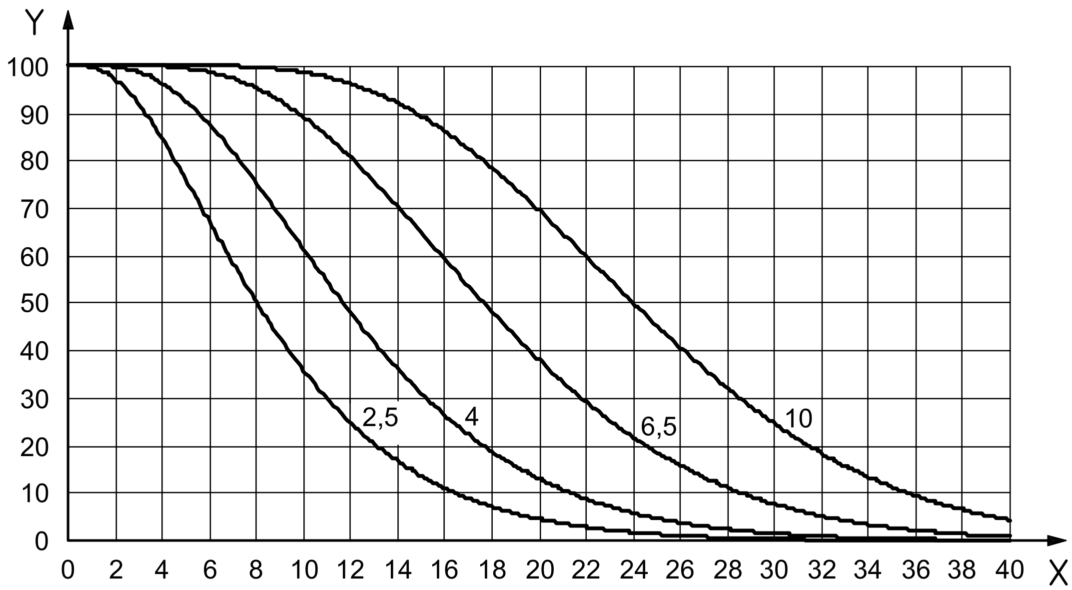
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter F										P_a
	0,65		1,0	1,5	2,5	4,0	6,5		10,0		
99,0	0,19	0,23	0,31	0,56	1,01	2,10	4,37	6,80	9,49	12,85	99,0
95,0	0,51	0,61	0,78	1,27	2,08	3,80	7,01	10,18	13,52	17,53	95,0
90,0	0,84	0,98	1,22	1,90	2,95	5,08	8,84	12,41	16,08	20,42	90,0
75,0	1,79	2,03	2,43	3,49	5,04	7,93	12,63	16,84	21,02	25,83	75,0
50,0	3,72	4,12	4,75	6,35	8,52	12,29	17,97	22,78	27,40	32,59	50,0
25,0	7,00	7,58	8,48	10,65	13,45	18,01	24,48	29,72	34,60	39,97	25,0
10,0	11,40	12,14	13,27	15,91	19,19	24,30	31,22	36,63	41,59	46,94	10,0
5,0	14,75	15,57	16,83	19,70	23,19	28,52	35,57	41,00	45,90	51,16	5,0
1,0	22,46	23,39	24,81	27,96	31,67	37,15	44,15	49,40	54,07	59,00	1,0
	1,0		1,5	2,5	4,0	6,5	10,0				
	Acceptance quality limit (tightened inspection) — Sample-size code letter F										
	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5		10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter H										

Figure 9 *(continued)*

24.6 Chart G



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 10 — Chart G — Operating characteristics curves for single sample plans, normal inspection

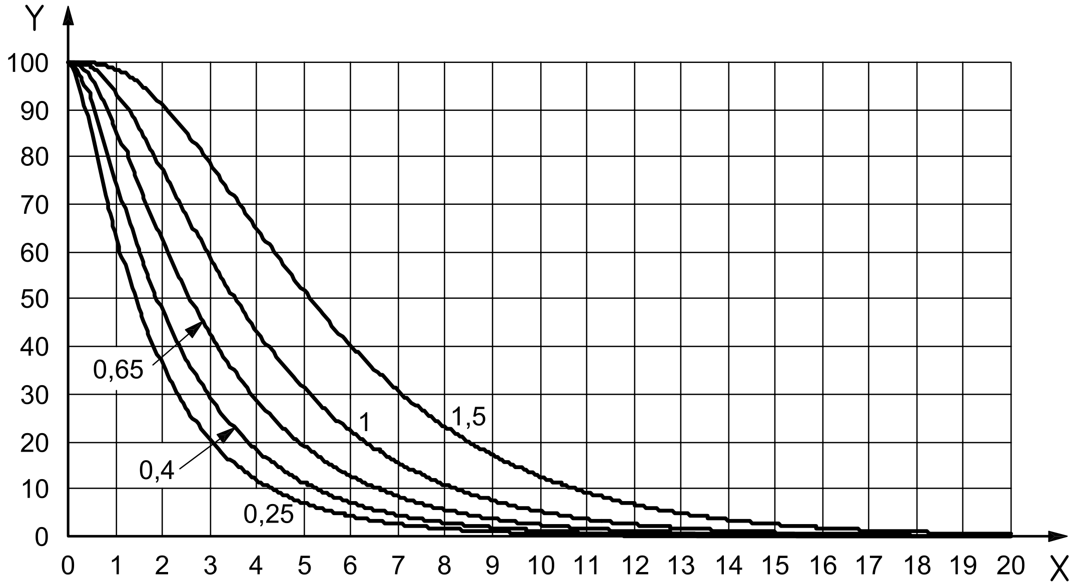
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

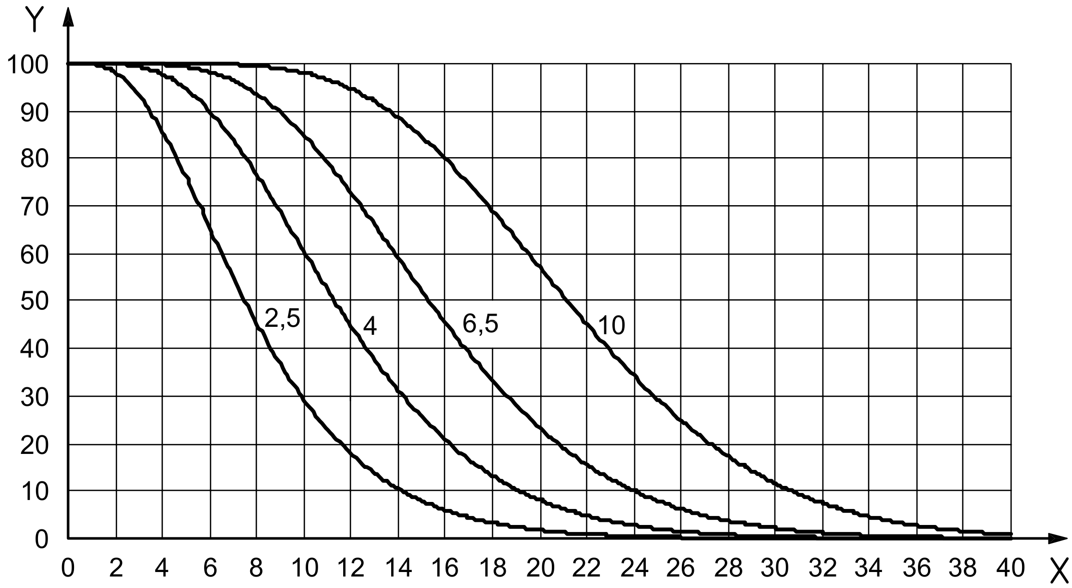
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter G												P_a
	0,40		0,65	1,0	1,5	2,5	4,0		6,5		10,0		
99,0	0,14	0,17	0,22	0,39	0,69	1,35	2,65	4,01	5,48	7,30	9,20	11,26	99,0
95,0	0,35	0,42	0,53	0,86	1,38	2,43	4,33	6,15	8,05	10,31	12,60	15,02	95,0
90,0	0,56	0,66	0,82	1,26	1,94	3,25	5,51	7,61	9,74	12,24	14,73	17,33	90,0
75,0	1,15	1,31	1,58	2,28	3,28	5,11	8,01	10,58	13,10	15,98	18,78	21,66	75,0
50,0	2,34	2,60	3,03	4,11	5,55	8,01	11,67	14,74	17,65	20,90	23,99	27,11	50,0
25,0	4,37	4,77	5,40	6,90	8,84	11,95	16,32	19,83	23,06	26,58	29,87	33,16	25,0
10,0	7,15	7,68	8,51	10,43	12,80	16,44	21,34	25,15	28,58	32,26	35,65	39,00	10,0
5,0	9,33	9,94	10,89	13,04	15,64	19,56	24,71	28,64	32,15	35,87	39,28	42,61	5,0
1,0	14,54	15,29	16,42	18,95	21,90	26,20	31,65	35,69	39,23	42,94	46,28	49,53	1,0
	0,65		1,0	1,5	2,5	4,0	6,5		10,0				
	Acceptance quality limit (tightened inspection) — Sample-size code letter G												
	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0		6,5		10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter J												

Figure 10 *(continued)*

24.7 Chart H



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 11 — Chart H — Operating characteristics curves for single sample plans, normal inspection

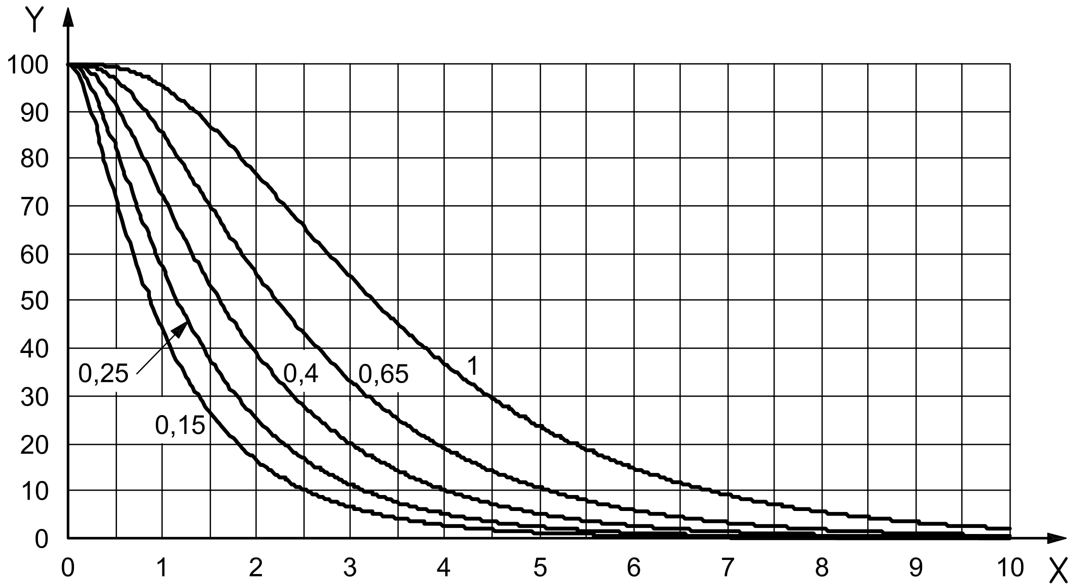
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

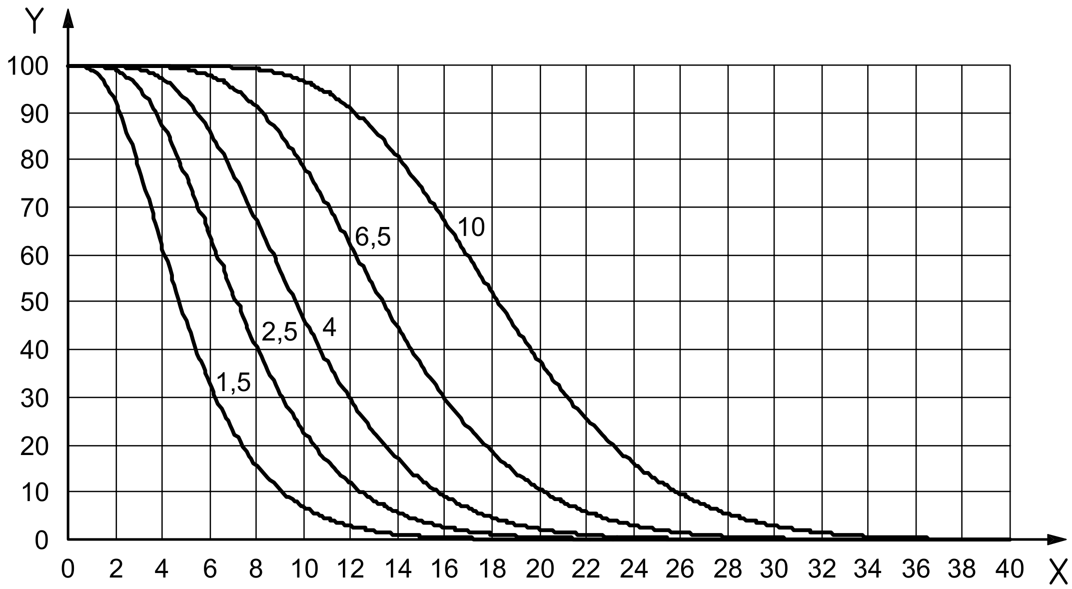
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter H													P_a
	0,25		0,40	0,65	1,0	1,5	2,5		4,0		6,5		10,0	
99,0	0,10	0,12	0,16	0,28	0,47	0,89	1,67	2,45	3,29	4,31	5,37	6,50	8,98	99,0
95,0	0,24	0,29	0,36	0,58	0,91	1,57	2,73	3,81	4,91	6,22	7,55	8,92	11,85	95,0
90,0	0,37	0,44	0,54	0,83	1,27	2,09	3,48	4,74	6,00	7,47	8,94	10,45	13,62	90,0
75,0	0,73	0,84	1,01	1,46	2,11	3,27	5,10	6,67	8,21	9,96	11,67	13,39	16,95	75,0
50,0	1,44	1,62	1,90	2,60	3,55	5,14	7,50	9,46	11,31	13,35	15,32	17,26	21,19	50,0
25,0	2,65	2,92	3,34	4,34	5,65	7,73	10,65	12,98	15,11	17,43	19,61	21,74	25,97	25,0
10,0	4,33	4,70	5,27	6,58	8,23	10,76	14,17	16,79	19,16	21,68	24,02	26,27	30,69	10,0
5,0	5,66	6,10	6,76	8,27	10,13	12,91	16,59	19,37	21,84	24,46	26,87	29,18	33,67	5,0
1,0	8,95	9,51	10,35	12,21	14,44	17,65	21,74	24,75	27,38	30,13	32,62	34,98	39,52	1,0
	0,40		0,65	1,0	1,5	2,5	4,0		6,5			10,0		
	Acceptance quality limit (tightened inspection) — Sample-size code letter H													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5		4,0		6,5	10,0	
	Acceptance quality limit (reduced inspection) — Sample-size code letter K													

Figure 11 *(continued)*

24.8 Chart J



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 12 — Chart J — Operating characteristics curves for single sample plans, normal inspection

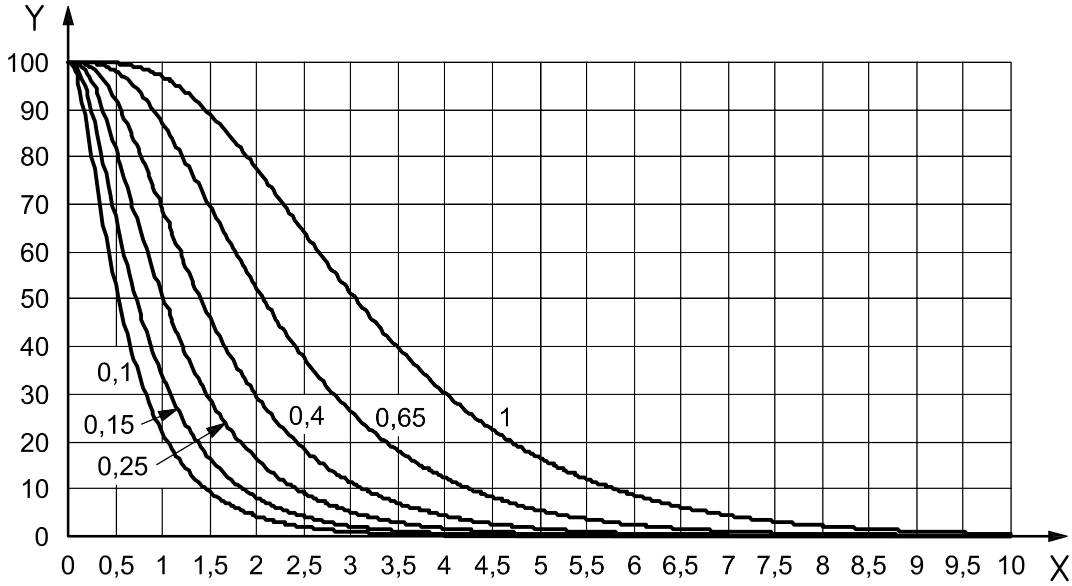
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

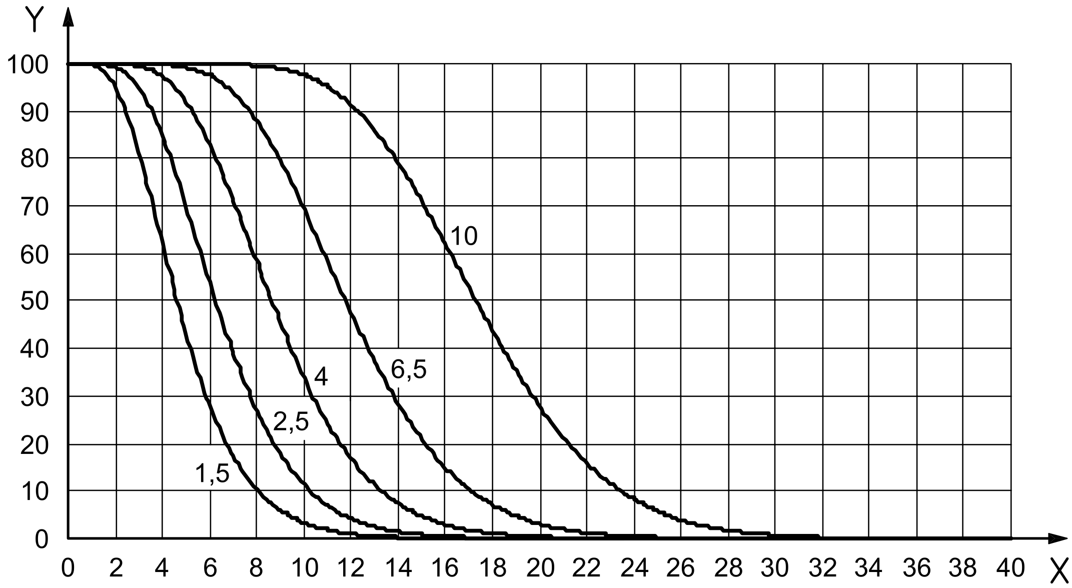
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter J														P_a
	0,15		0,25	0,40	0,65	1,0	1,5		2,5		4,0		6,5		10,0	
99,0	0,07	0,09	0,12	0,20	0,33	0,59	1,08	1,55	2,04	2,63	3,24	3,88	5,28	6,79	8,42	99,0
95,0	0,16	0,19	0,25	0,39	0,61	1,03	1,74	2,39	3,06	3,83	4,60	5,40	7,11	8,90	10,78	95,0
90,0	0,24	0,29	0,36	0,55	0,83	1,35	2,21	2,98	3,74	4,62	5,49	6,38	8,26	10,20	12,21	90,0
75,0	0,46	0,53	0,64	0,93	1,35	2,08	3,22	4,20	5,16	6,22	7,26	8,31	10,47	12,66	14,89	75,0
50,0	0,87	0,99	1,17	1,62	2,22	3,25	4,75	5,99	7,17	8,45	9,68	10,91	13,38	15,82	18,28	50,0
25,0	1,57	1,74	2,01	2,67	3,51	4,88	6,79	8,30	9,71	11,21	12,62	14,01	16,77	19,45	22,10	25,0
10,0	2,53	2,78	3,15	4,02	5,12	6,82	9,11	10,87	12,48	14,17	15,73	17,26	20,24	23,09	25,88	10,0
5,0	3,31	3,60	4,04	5,06	6,31	8,22	10,73	12,64	14,36	16,15	17,80	19,39	22,49	25,43	28,28	5,0
1,0	5,25	5,63	6,21	7,52	9,07	11,37	14,29	16,44	18,34	20,30	22,08	23,79	27,05	30,10	33,04	1,0
	0,25		0,40	0,65	1,0	1,5	25		4,0			6,5		10,0		
	Acceptance quality limit (tightened inspection) — Sample-size code letter J															
	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5		2,5		4,0	6,5			
	Acceptance quality limit (reduced inspection) — Sample-size code letter L															

Figure 12 *(continued)*

24.9 Chart K



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 13 — Chart K —Operating characteristics curves for single sample plans, normal inspection

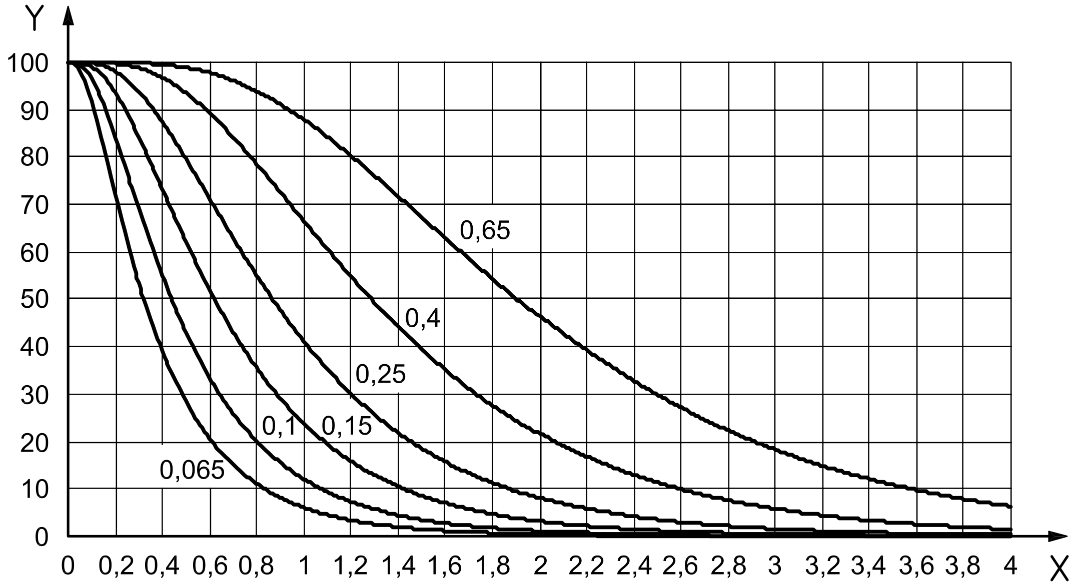
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

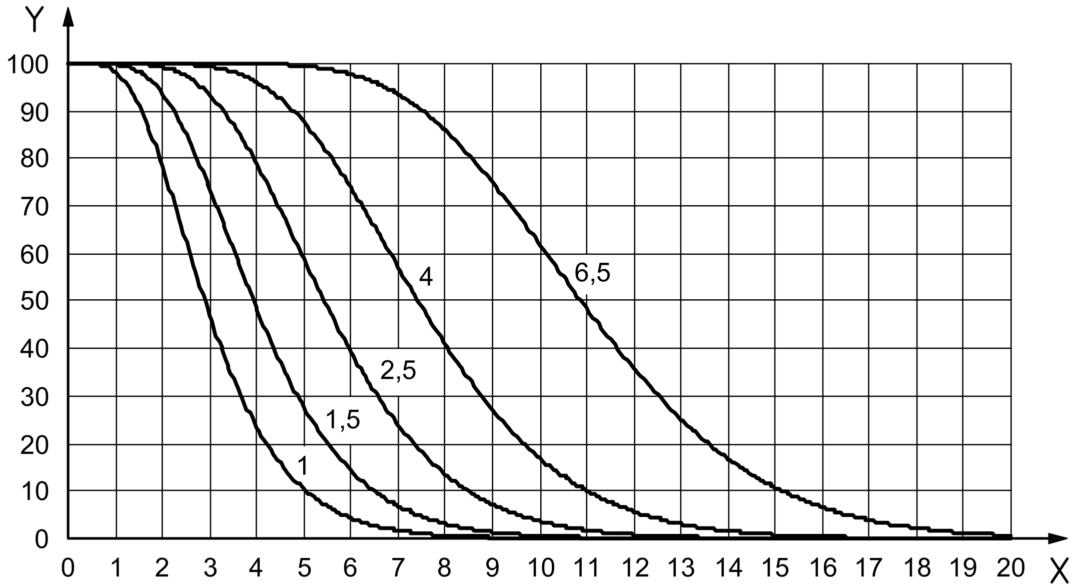
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter K																	P_a
	0,10		0,15	0,25	0,40	0,65	1,0		1,5		2,5		4,0		6,5		10,0	
99,0	0,06	0,07	0,09	0,15	0,24	0,42	0,74	1,05	1,36	1,74	2,12	2,51	3,36	4,28	5,24	7,33	8,99	99,0
95,0	0,12	0,14	0,17	0,27	0,42	0,70	1,17	1,59	2,01	2,50	2,98	3,48	4,53	5,63	6,76	9,18	11.05	95,0
90,0	0,17	0,19	0,24	0,37	0,56	0,90	1,46	1,96	2,44	3,00	3,55	4,11	5,27	6,47	7,70	10,29	12,28	90,0
75,0	0,30	0,34	0,42	0,61	0,88	1,36	2,10	2,73	3,34	4,02	4,69	5,35	6,71	8,09	9,48	12,26	14,53	75,0
50,0	0,53	0,61	0,72	1,01	1,41	2,07	3,05	3,86	4,62	5,46	6,26	7,05	8,63	10,21	11,78	14,96	17,32	50,0
25,0	0,93	1,04	1,21	1,63	2,18	3,07	4,32	5,33	6,25	7,25	8,19	9,11	10,92	12,69	14,43	17,89	20,42	25,0
10,0	1,46	1,62	1,86	2,42	3,13	4,25	5,78	6,98	8,05	9,20	10,26	11,30	13,30	15,24	17,12	20,80	23,46	10,0
5,0	1,89	2,08	2,36	3,02	3,85	5,12	6,82	8,12	9,29	10,52	11,66	12,76	14,88	16,91	18,86	22,67	25,38	5,0
1,0	2,97	3,22	3,60	4,47	5,52	7,08	9,11	10,63	11,97	13,35	14,62	15,84	18,14	20,32	22,39	26,39	29,20	1,0
	0,15		0,25	0,40	0,65	1,0	1,5		2,5			4,0		6,5		10,0		
	Acceptance quality limit (tightened inspection) — Sample-size code letter K																	
	0,04	0,065	0,010	0,15	0,25	0,40	0,65	1,0		1,5		2,5	4,0					
	Acceptance quality limit (reduced inspection) — Sample-size code letter M																	

Figure 13 *(continued)*

24.10 Chart L



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 14 — Chart L — Operating characteristics curves for single sample plans, normal inspection

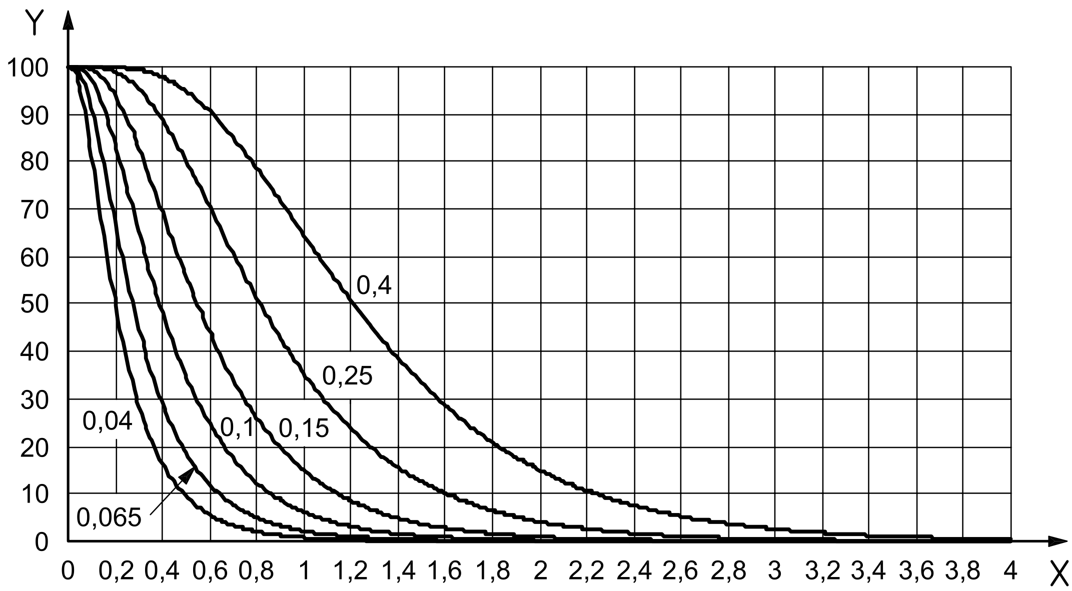
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

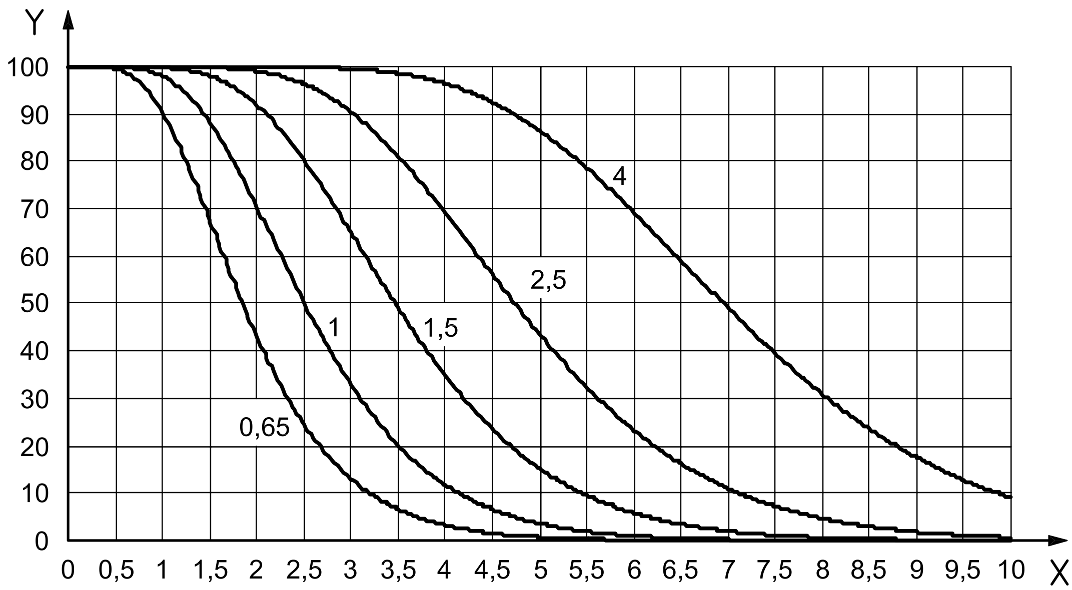
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter L																	P_a
	0,065		0,10	0,15	0,25	0,40	0,65		1,0		1,5		2,5		4,0		6,5	
99,0	0,04	0,05	0,06	0,10	0,17	0,29	0,50	0,69	0,89	1,12	1,35	1,59	2,10	2,65	3,22	4,44	5,41	99,0
95,0	0,08	0,09	0,12	0,18	0,28	0,46	0,76	1,02	1,29	1,59	1,89	2,19	2,83	3,50	4,18	5,61	6,73	95,0
90,0	0,11	0,13	0,16	0,24	0,36	0,59	0,94	1,25	1,55	1,90	2,24	2,58	3,30	4,03	4,78	6,33	7,52	90,0
75,0	0,19	0,22	0,26	0,39	0,56	0,86	1,33	1,73	2,11	2,54	2,95	3,36	4,21	5,06	5,93	7,67	9,00	75,0
50,0	0,32	0,37	0,44	0,62	0,87	1,29	1,91	2,42	2,90	3,43	3,93	4,43	5,44	6,43	7,43	9,40	10,88	50,0
25,0	0,54	0,61	0,72	0,98	1,32	1,88	2,68	3,33	3,92	4,56	5,16	5,75	6,92	8,06	9,19	11,39	13,01	25,0
10,0	0,84	0,93	1,08	1,43	1,88	2,59	3,58	4,35	5,05	5,80	6,49	7,17	8,49	9,76	11,00	13,40	15,15	10,0
5,0	1,07	1,18	1,36	1,77	2,29	3,11	4,21	5,07	5,83	6,65	7,40	8,13	9,54	10,89	12,20	14,72	16,53	5,0
1,0	1,66	1,81	2,05	2,60	3,27	4,30	5,64	6,66	7,55	8,49	9,35	10,17	11,75	13,25	14,68	17,39	19,32	1,0
	0,10		0,15	0,25	0,40	0,65	1,0		1,5			2,5		4,0		6,5		
	Acceptance quality limit (tightened inspection) — Sample-size code letter L																	
	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65		1,0		1,5	2,5					
	Acceptance quality limit (reduced inspection) — Sample-size code letter N																	

Figure 14 *(continued)*

24.11 Chart M



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 15 — Chart M — Operating characteristics curves for single sample plans, normal inspection

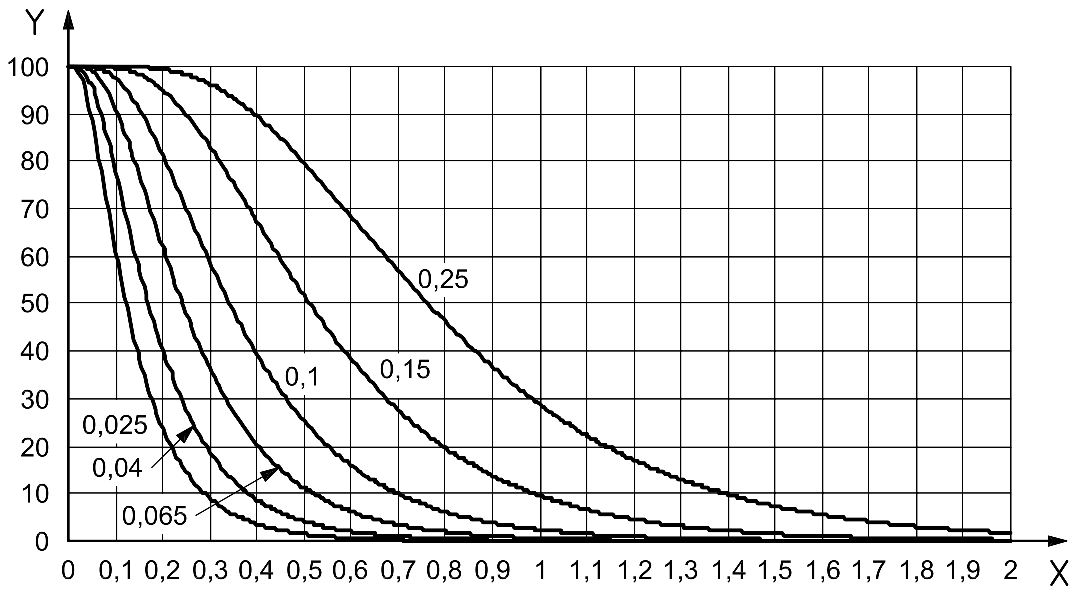
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

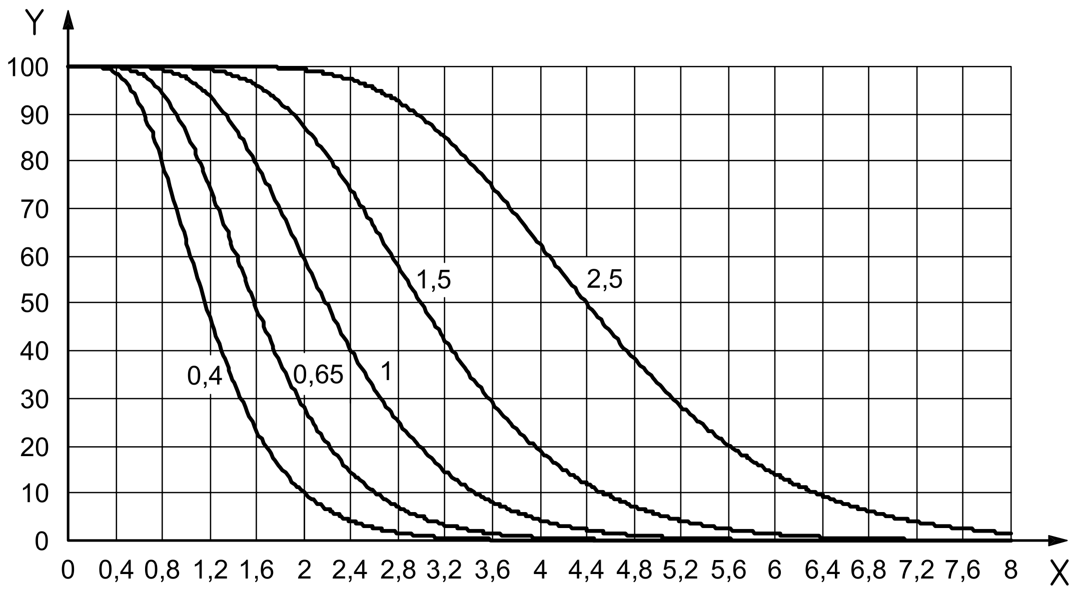
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter M																	P_a
	0,04		0,065	0,10	0,15	0,25	0,40		0,65		1,0		1,5		2,5		4,0	
99,0	0,03	0,04	0,05	0,07	0,12	0,20	0,33	0,46	0,59	0,73	0,88	1,03	1,35	1,68	2,04	2,78	3,36	99,0
95,0	0,05	0,06	0,08	0,12	0,19	0,31	0,50	0,67	0,84	1,03	1,22	1,41	1,81	2,22	2,65	3,53	4,21	95,0
90,0	0,07	0,09	0,11	0,16	0,24	0,38	0,61	0,81	1,01	1,23	1,44	1,66	2,10	2,56	3,03	3,99	4,73	90,0
75,0	0,12	0,14	0,17	0,25	0,36	0,55	0,85	1,11	1,35	1,63	1,89	2,15	2,68	3,22	3,76	4,87	5,70	75,0
50,0	0,20	0,23	0,27	0,39	0,55	0,82	1,21	1,54	1,85	2,19	2,51	2,83	3,47	4,10	4,74	6,01	6,95	50,0
25,0	0,32	0,37	0,43	0,60	0,82	1,17	1,69	2,10	2,48	2,90	3,29	3,67	4,42	5,16	5,90	7,34	8,39	25,0
10,0	0,49	0,55	0,64	0,86	1,14	1,60	2,24	2,74	3,20	3,69	4,14	4,58	5,45	6,28	7,11	8,70	9,86	10,0
5,0	0,62	0,69	0,80	1,06	1,39	1,91	2,63	3,19	3,69	4,23	4,73	5,21	6,14	7,03	7,92	9,60	10,82	5,0
1,0	0,95	1,05	1,19	1,54	1,97	2,63	3,51	4,19	4,79	5,42	5,99	6,55	7,61	8,62	9,60	11,46	12,79	1,0
	0,065		0,10	0,15	0,25	0,40	0,65		1,0			1,5		2,5		4,0		
	Acceptance quality limit (tightened inspection) — Sample-size code letter M																	
	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40		0,65		1,0	1,5					
	Acceptance quality limit (reduced inspection) — Sample-size code letter P																	

Figure 15 *(continued)*

24.12 Chart N



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 16 — Chart N — Operating characteristics curves for single sample plans, normal inspection

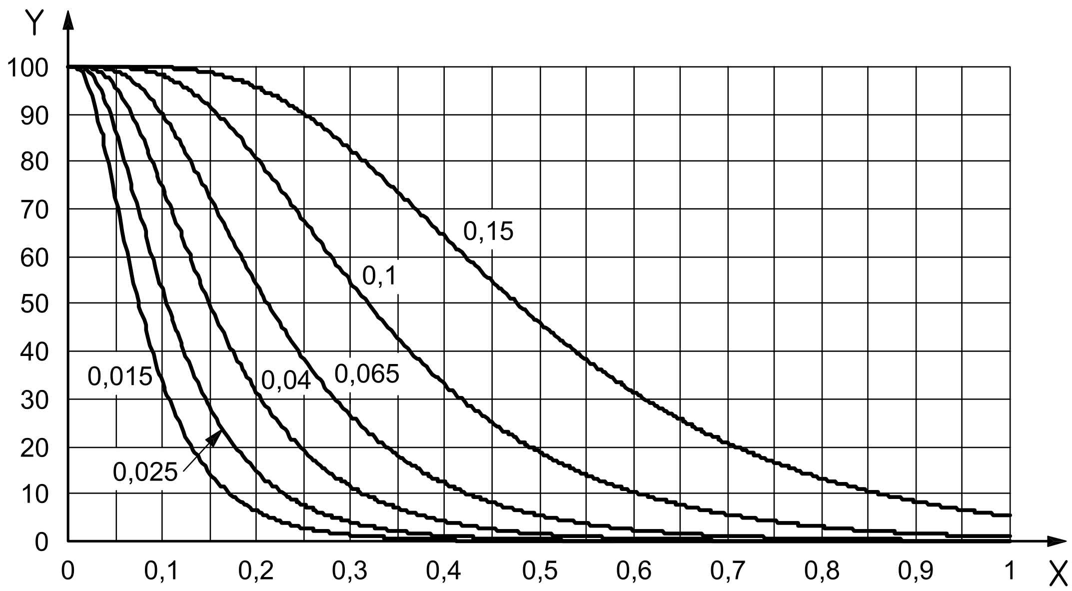
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

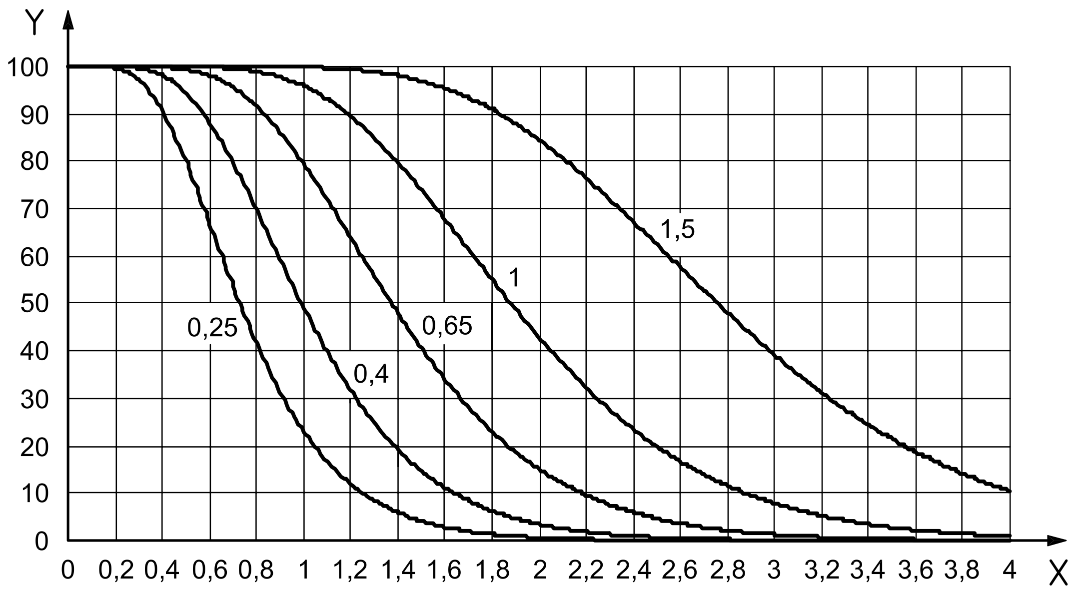
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter N																	P_a
	0,025		0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0		1,5		2,5	
99,0	0,02	0,02	0,03	0,05	0,08	0,13	0,22	0,30	0,38	0,47	0,56	0,66	0,85	1,06	1,28	1,73	2,08	99,0
95,0	0,04	0,04	0,05	0,08	0,12	0,20	0,32	0,43	0,54	0,66	0,78	0,90	1,14	1,40	1,66	2,20	2,63	95,0
90,0	0,05	0,06	0,07	0,10	0,16	0,25	0,39	0,52	0,64	0,78	0,91	1,05	1,33	1,61	1,90	2,50	2,96	90,0
75,0	0,08	0,09	0,11	0,16	0,23	0,35	0,54	0,70	0,86	1,03	1,19	1,36	1,69	2,03	2,37	3,06	3,58	75,0
50,0	0,12	0,14	0,17	0,24	0,34	0,51	0,76	0,97	1,17	1,38	1,58	1,79	2,19	2,59	3,00	3,80	4,40	50,0
25,0	0,19	0,22	0,26	0,37	0,50	0,73	1,06	1,32	1,56	1,83	2,07	2,32	2,80	3,28	3,75	4,67	5,36	25,0
10,0	0,29	0,33	0,38	0,52	0,70	0,99	1,39	1,71	2,01	2,32	2,61	2,90	3,46	4,01	4,54	5,58	6,34	10,0
5,0	0,36	0,41	0,47	0,64	0,84	1,18	1,63	1,99	2,32	2,67	2,99	3,30	3,91	4,50	5,07	6,18	6,99	5,0
1,0	0,55	0,61	0,70	0,92	1,19	1,61	2,18	2,61	3,01	3,43	3,80	4,17	4,87	5,55	6,20	7,44	8,34	1,0
	0,04		0,065	0,10	0,15	0,25	0,40		0,65			1,0		1,5		2,5		
	Acceptance quality limit (tightened inspection) — Sample-size code letter N																	
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65	1,0					
	Acceptance quality limit (reduced inspection) — Sample-size code letter Q																	

Figure 16 *(continued)*

24.13 Chart P



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 17 — Chart P — Operating characteristics curves for single sample plans, normal inspection

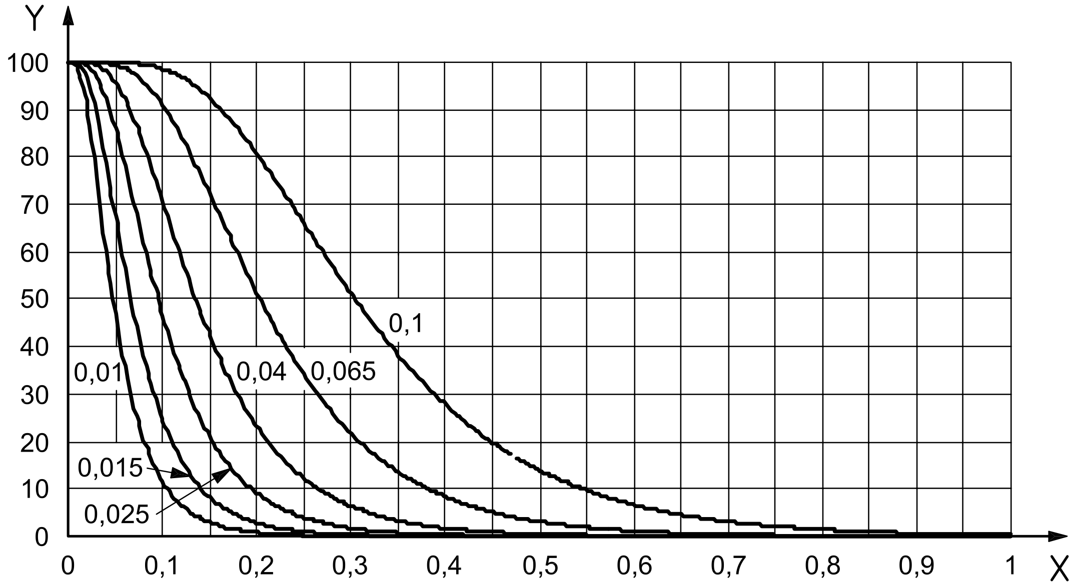
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

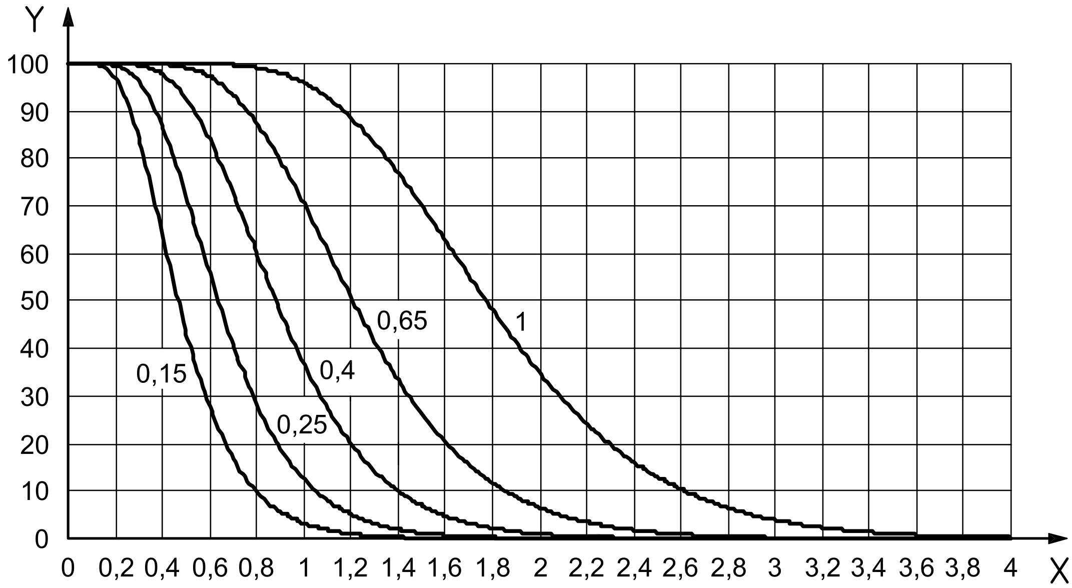
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter P																	P_a
	0,015		0,025	0,040	0,065	0,10	0,15		0,25		0,40		0,65		1,0		1,5	
99,0	0,01	0,02	0,02	0,03	0,05	0,09	0,14	0,19	0,24	0,30	0,36	0,42	0,54	0,67	0,80	1,07	1,28	99,0
95,0	0,02	0,03	0,03	0,05	0,08	0,13	0,21	0,27	0,34	0,42	0,49	0,56	0,72	0,88	1,04	1,37	1,62	95,0
90,0	0,03	0,04	0,04	0,07	0,10	0,16	0,25	0,33	0,41	0,49	0,58	0,66	0,83	1,01	1,19	1,55	1,83	90,0
75,0	0,05	0,06	0,07	0,10	0,14	0,22	0,34	0,44	0,54	0,65	0,75	0,85	1,06	1,27	1,48	1,91	2,23	75,0
50,0	0,08	0,09	0,10	0,15	0,21	0,32	0,48	0,61	0,73	0,86	0,99	1,12	1,37	1,63	1,88	2,38	2,76	50,0
25,0	0,12	0,13	0,16	0,22	0,31	0,45	0,66	0,82	0,97	1,14	1,30	1,45	1,76	2,07	2,36	2,95	3,38	25,0
10,0	0,17	0,19	0,23	0,31	0,42	0,61	0,86	1,06	1,25	1,45	1,64	1,82	2,18	2,54	2,87	3,54	4,03	10,0
5,0	0,21	0,24	0,28	0,38	0,51	0,72	1,01	1,24	1,45	1,67	1,87	2,08	2,47	2,85	3,22	3,94	4,46	5,0
1,0	0,32	0,36	0,41	0,55	0,71	0,98	1,35	1,63	1,88	2,15	2,39	2,63	3,09	3,54	3,96	4,78	5,37	1,0
	0,025		0,04	0,065	0,10	0,15	0,25		0,40			0,65		1,0		1,5		
	Acceptance quality limit (tightened inspection) — Sample-size code letter P																	
		0,01	0,015	0,025	0,04	0,065	0,10	0,15		0,25		0,40	0,65					
	Acceptance quality limit (reduced inspection) — Sample-size code letter R																	

Figure 17 *(continued)*

24.14 Chart Q



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 18 — Chart Q — Operating characteristics curves for single sample plans, normal inspection

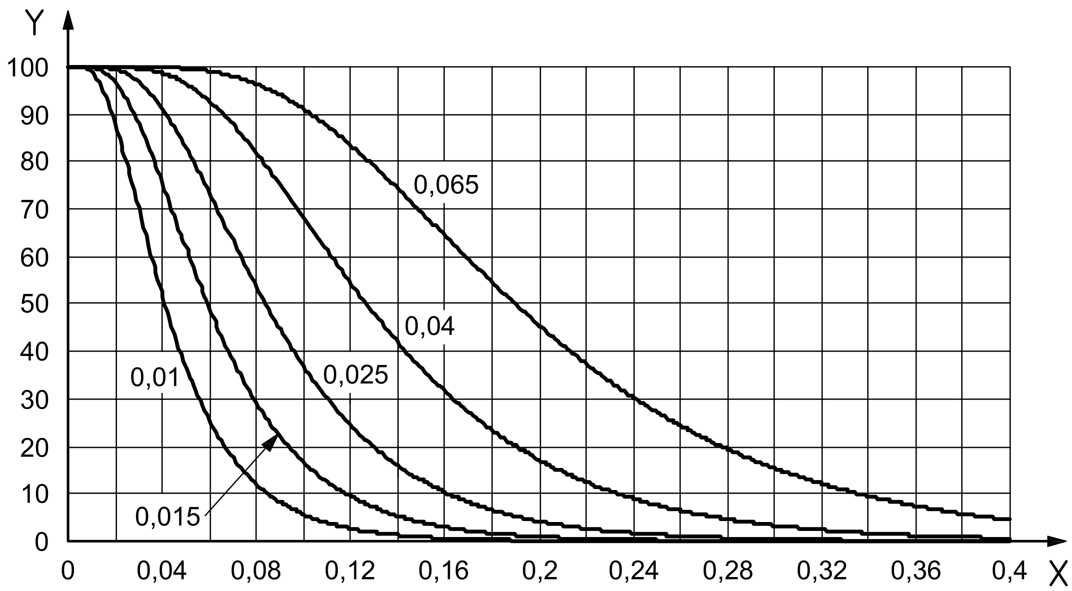
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

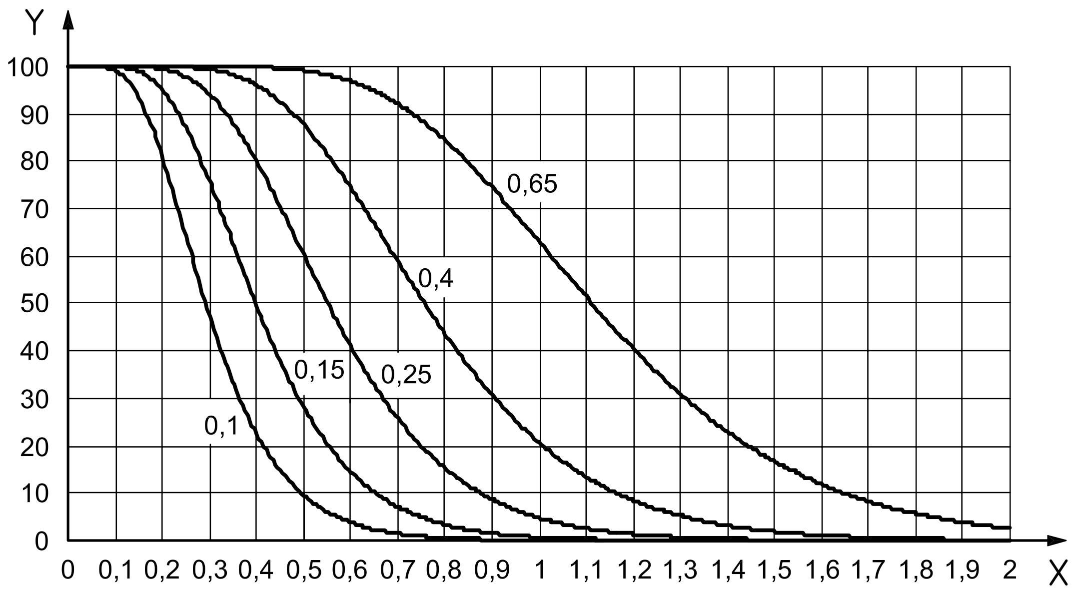
P_a	Acceptance quality limit (normal inspection) — Sample-size code letter Q														P_a
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0	
99,0	0,01	0,01	0,02	0,03	0,06	0,09	0,16	0,23	0,27	0,35	0,43	0,51	0,68	0,82	99,0
95,0	0,02	0,02	0,04	0,05	0,08	0,14	0,22	0,32	0,36	0,46	0,56	0,66	0,87	1,04	95,0
90,0	0,02	0,03	0,04	0,07	0,10	0,16	0,26	0,37	0,43	0,54	0,65	0,76	0,99	1,17	90,0
75,0	0,03	0,04	0,06	0,09	0,14	0,22	0,35	0,48	0,55	0,68	0,82	0,95	1,22	1,43	75,0
50,0	0,05	0,07	0,10	0,14	0,20	0,30	0,47	0,64	0,72	0,88	1,05	1,21	1,53	1,77	50,0
25,0	0,07	0,10	0,14	0,19	0,28	0,42	0,62	0,83	0,93	1,13	1,33	1,52	1,90	2,18	25,0
10,0	0,11	0,14	0,19	0,27	0,38	0,55	0,80	1,05	1,17	1,40	1,63	1,86	2,29	2,61	10,0
5,0	0,13	0,17	0,24	0,32	0,45	0,64	0,92	1,20	1,33	1,59	1,84	2,08	2,56	2,90	5,0
1,0	0,19	0,25	0,33	0,44	0,61	0,85	1,20	1,53	1,69	1,99	2,29	2,57	3,12	3,51	1,0
	0,015	0,025	0,04	0,065	0,10	0,15	0,25		0,40		0,65		1,0		
	Acceptance quality limit (tightened inspection) — Sample-size code letter Q														

Figure 18 *(continued)*

24.15 Chart R



a)



b)

Key

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted (P_a)

NOTE Figures on curves are AQLs in percent nonconforming.

Figure 19 — Chart R — Operating characteristics curves for single sample plans, normal inspection

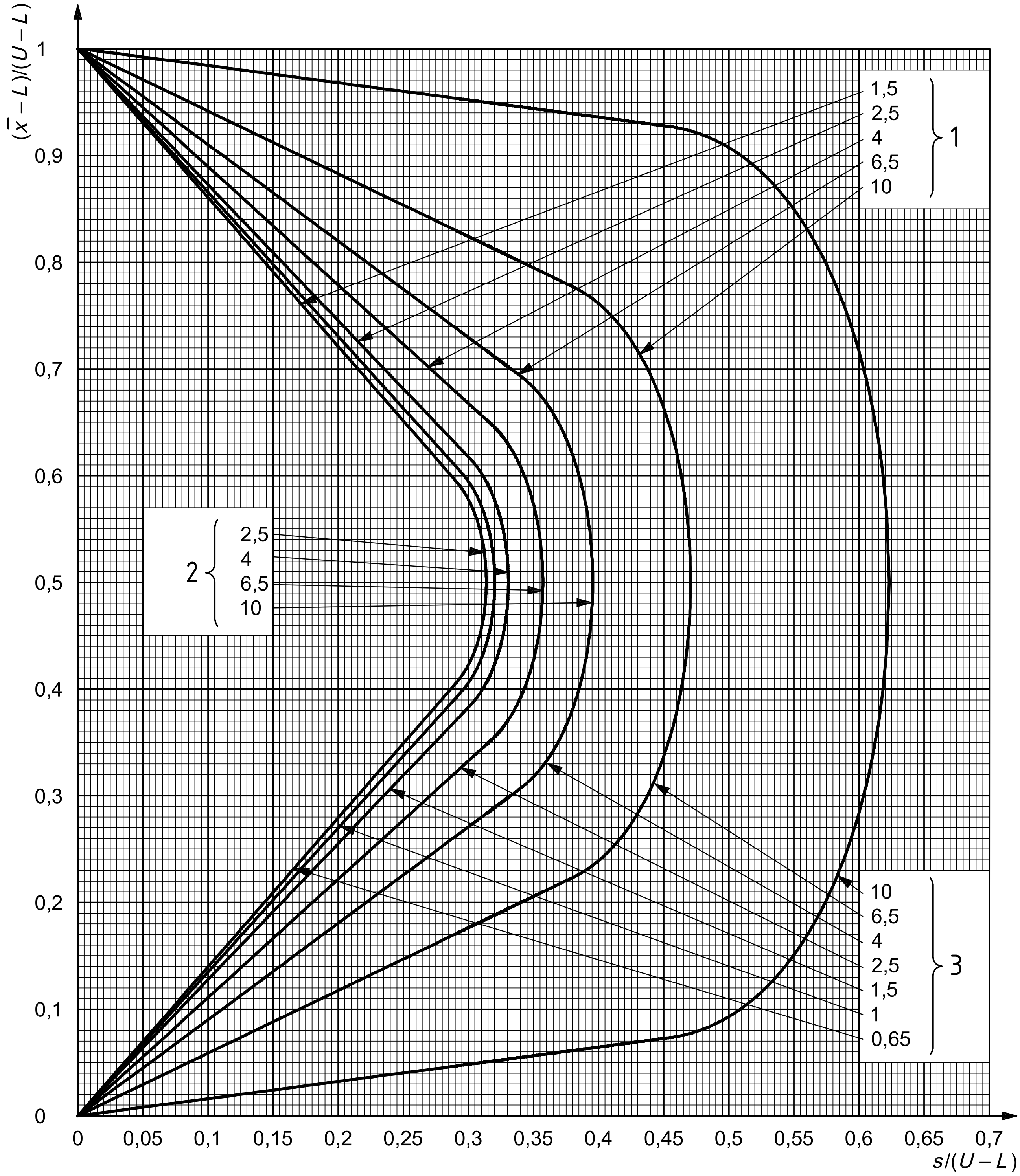
Tabulated values for operating characteristic curves for single sampling plans

All data in percent

P_a	Acceptance quality limit (normal inspection) — Sample-size code letter R														P_a
		0,010	0,015	0,025	0,04	0,065	0,10	0,15		0,25		0,40		0,65	
99,0	0,01	0,01	0,01	0,02	0,04	0,06	0,10	0,15	0,17	0,22	0,27	0,32	0,43	0,51	99,0
95,0	0,01	0,01	0,02	0,03	0,05	0,09	0,14	0,20	0,23	0,29	0,35	0,42	0,55	0,65	95,0
90,0	0,01	0,02	0,03	0,04	0,07	0,10	0,17	0,23	0,27	0,34	0,41	0,48	0,62	0,73	90,0
75,0	0,02	0,03	0,04	0,06	0,09	0,14	0,22	0,30	0,34	0,43	0,51	0,60	0,77	0,90	75,0
50,0	0,03	0,04	0,06	0,08	0,13	0,19	0,29	0,40	0,45	0,55	0,65	0,76	0,96	1,11	50,0
25,0	0,04	0,06	0,09	0,12	0,18	0,26	0,39	0,52	0,58	0,71	0,83	0,95	1,19	1,37	25,0
10,0	0,06	0,08	0,12	0,16	0,23	0,34	0,49	0,65	0,73	0,88	1,02	1,16	1,44	1,65	10,0
5,0	0,08	0,10	0,14	0,19	0,28	0,39	0,57	0,75	0,83	0,99	1,15	1,31	1,61	1,83	5,0
1,0	0,11	0,15	0,20	0,27	0,37	0,52	0,74	0,95	1,05	1,25	1,44	1,62	1,97	2,23	1,0
	0,010	0,015	0,025	0,04	0,065	0,10	0,15		0,25		0,40		0,65		
	Acceptance quality limit (tightened inspection) — Sample-size code letter R														

Figure 19 *(continued)*

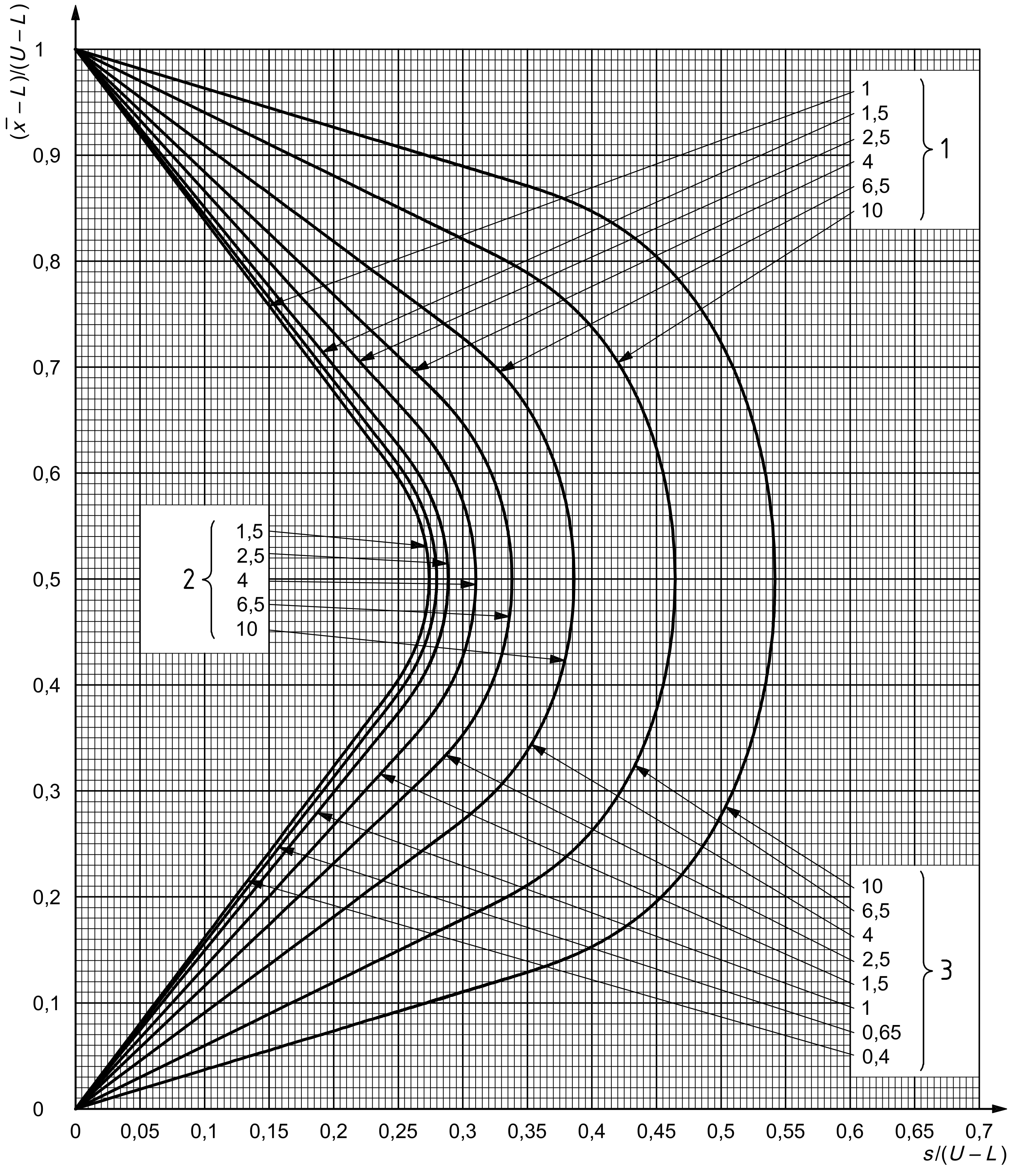
25 Charts *s*-D to *s*-R (Figures 20 to 32) — Acceptance curves for combined control of double specification limits: "*s*" method



Key

1 AQL % (normal, D)
2 AQL % (tightened, D)
3 AQL % (reduced, F)

Figure 20 — Chart *s*-D — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter D (sample size 6) (sample-size code letter F for reduced inspection)



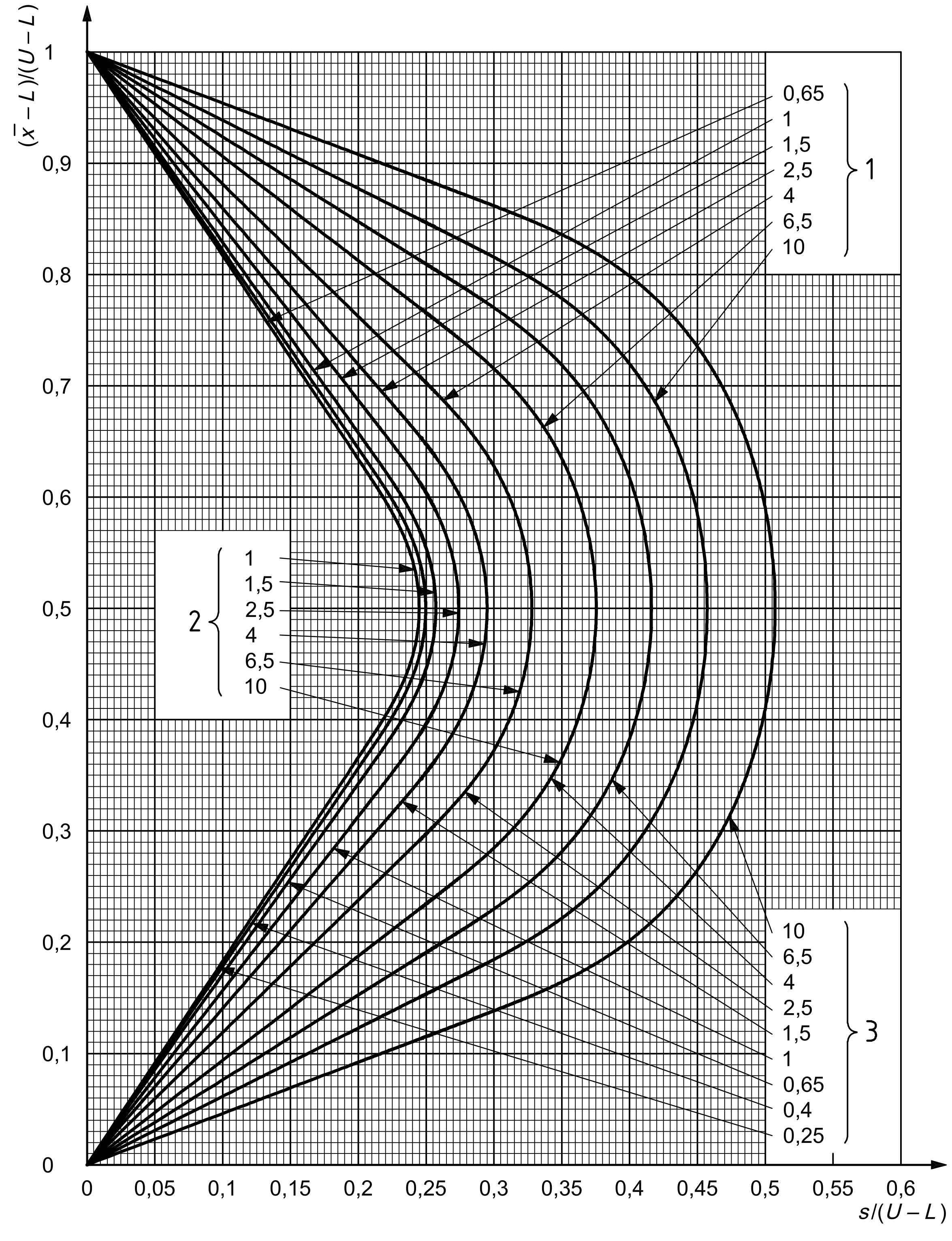
Key

1 AQL % (normal, E)

2 AQL % (tightened, E)

3 AQL % (reduced, G)

Figure 21 — Chart *s*-E — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter E (sample size 9) (sample-size code letter G for reduced inspection)



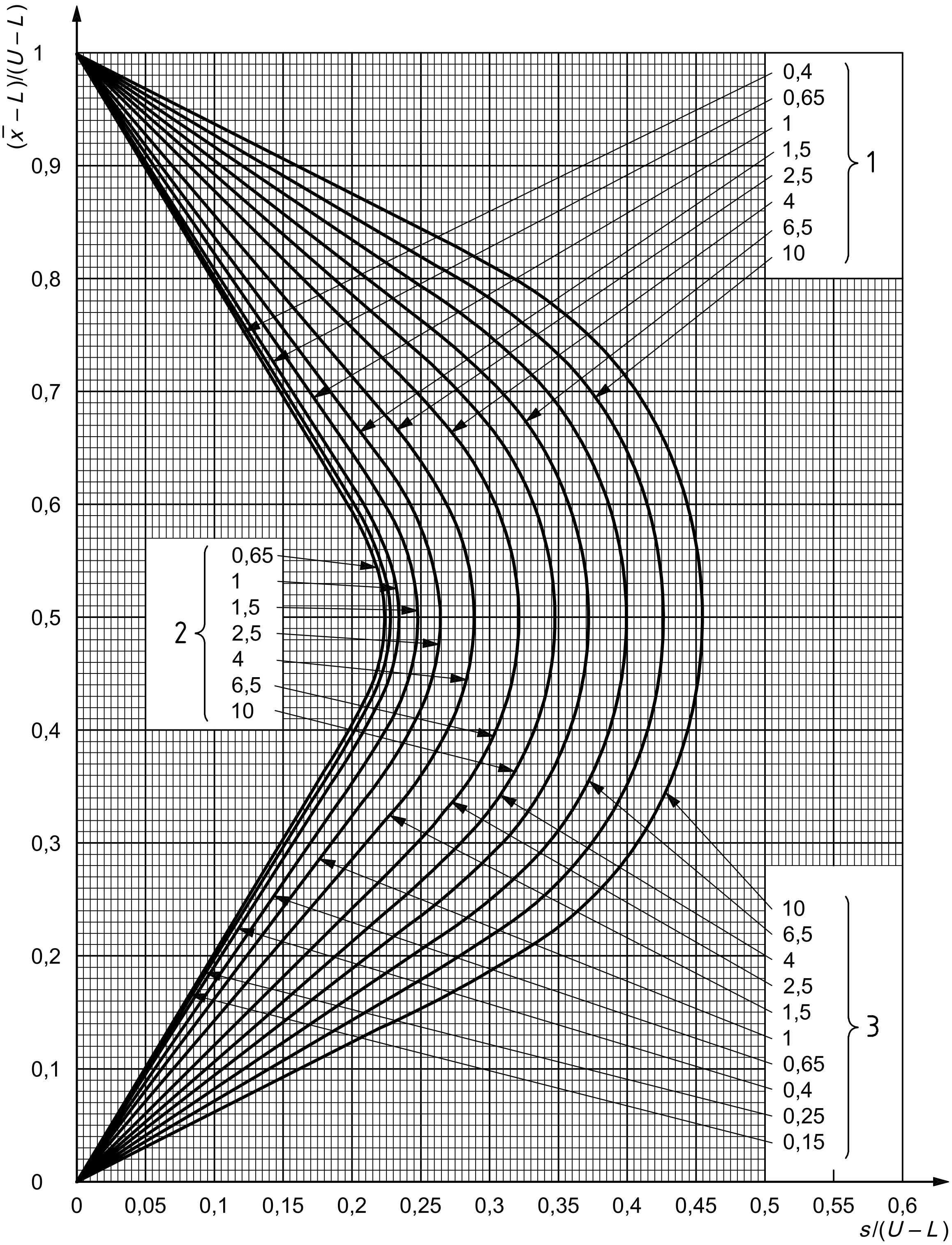
Key

1 AQL % (normal, F)

2 AQL % (tightened, F)

3 AQL % (reduced, H)

Figure 22 — Chart *s*-F — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter F (sample size 13) (sample-size code letter H for reduced inspection)



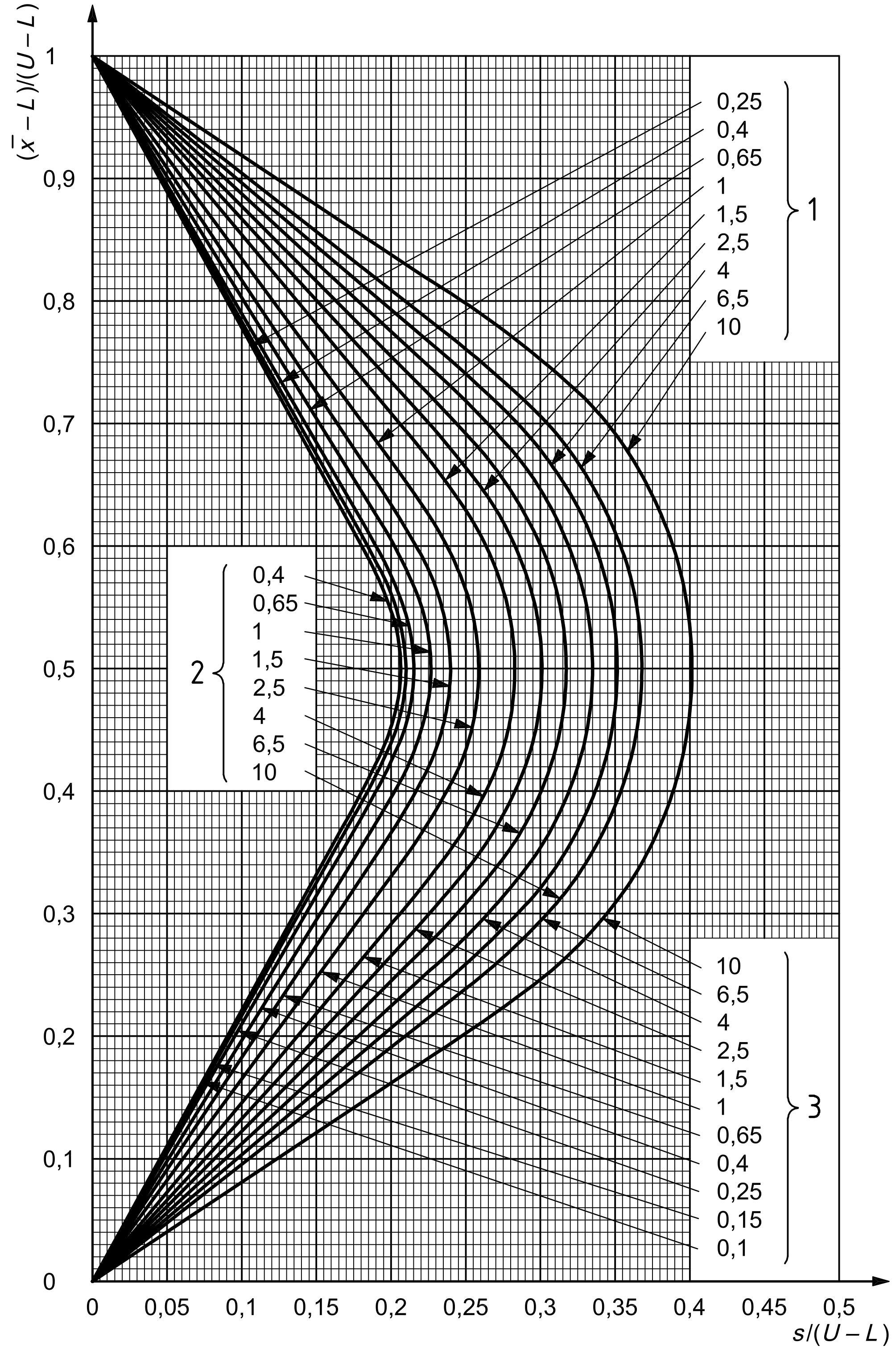
Key

1 AQL % (normal, G)

2 AQL % (tightened, G)

3 AQL % (reduced, J)

Figure 23 — Chart *s*-G — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter G (sample size 18) (sample-size code letter J for reduced inspection)



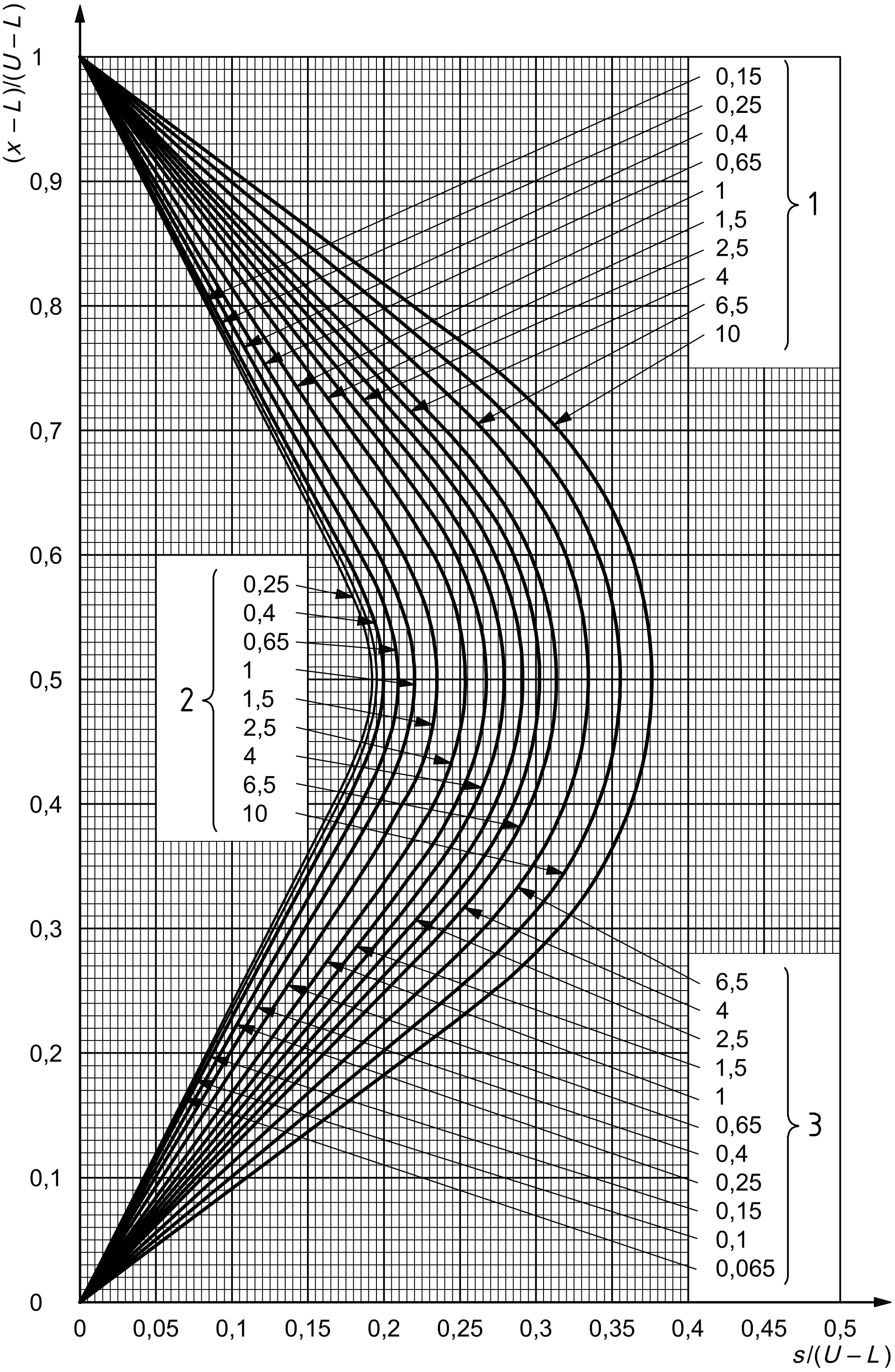
Key

1 AQL % (normal, H)

2 AQL % (tightened, H)

3 AQL % (reduced, K)

Figure 24 — Chart *s*-H — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter H (sample size 25) (sample-size code letter K for reduced inspection)



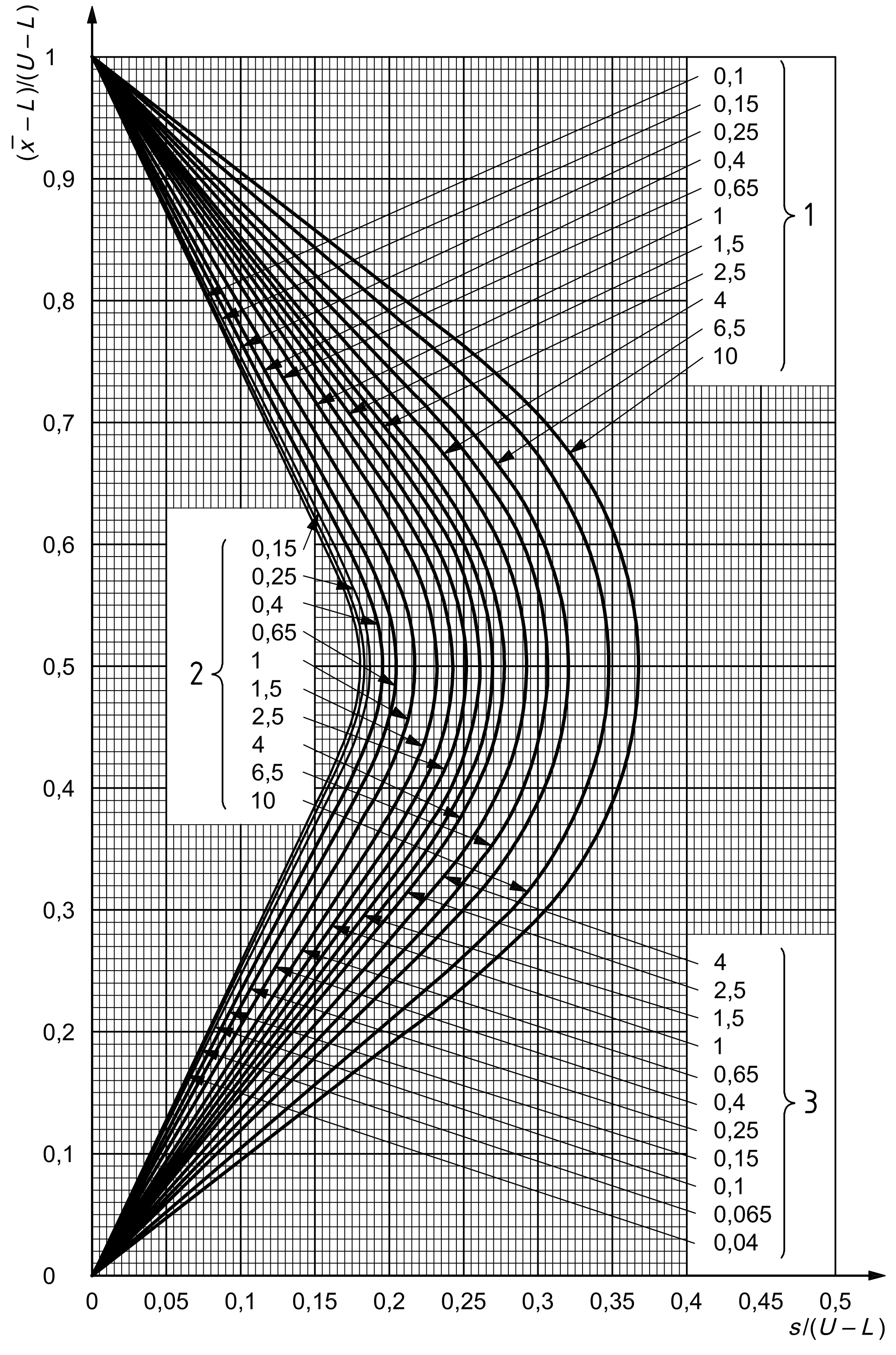
Key

1 AQL % (normal,J)

2 AQL % (tightened, J)

3 AQL % (reduced, L)

Figure 25 — Chart *s*-J — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter J (sample size 35) (sample-size code letter L for reduced inspection)



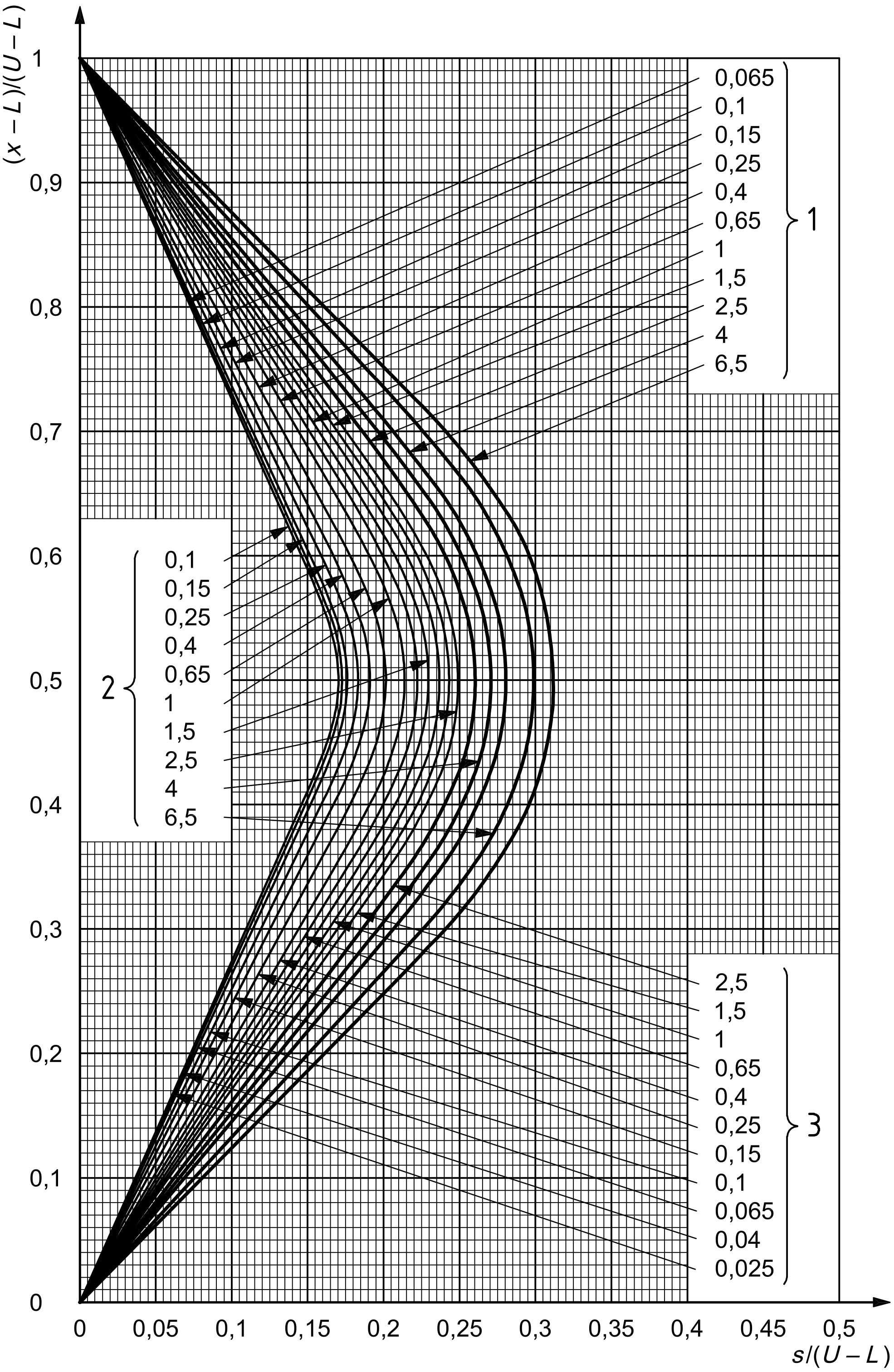
Key

1 AQL % (normal, K)

2 AQL % (tightened, K)

3 AQL % (reduced, M)

Figure 26 — Chart *s*-K — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter K (sample size 50) (sample-size code letter M for reduced inspection)



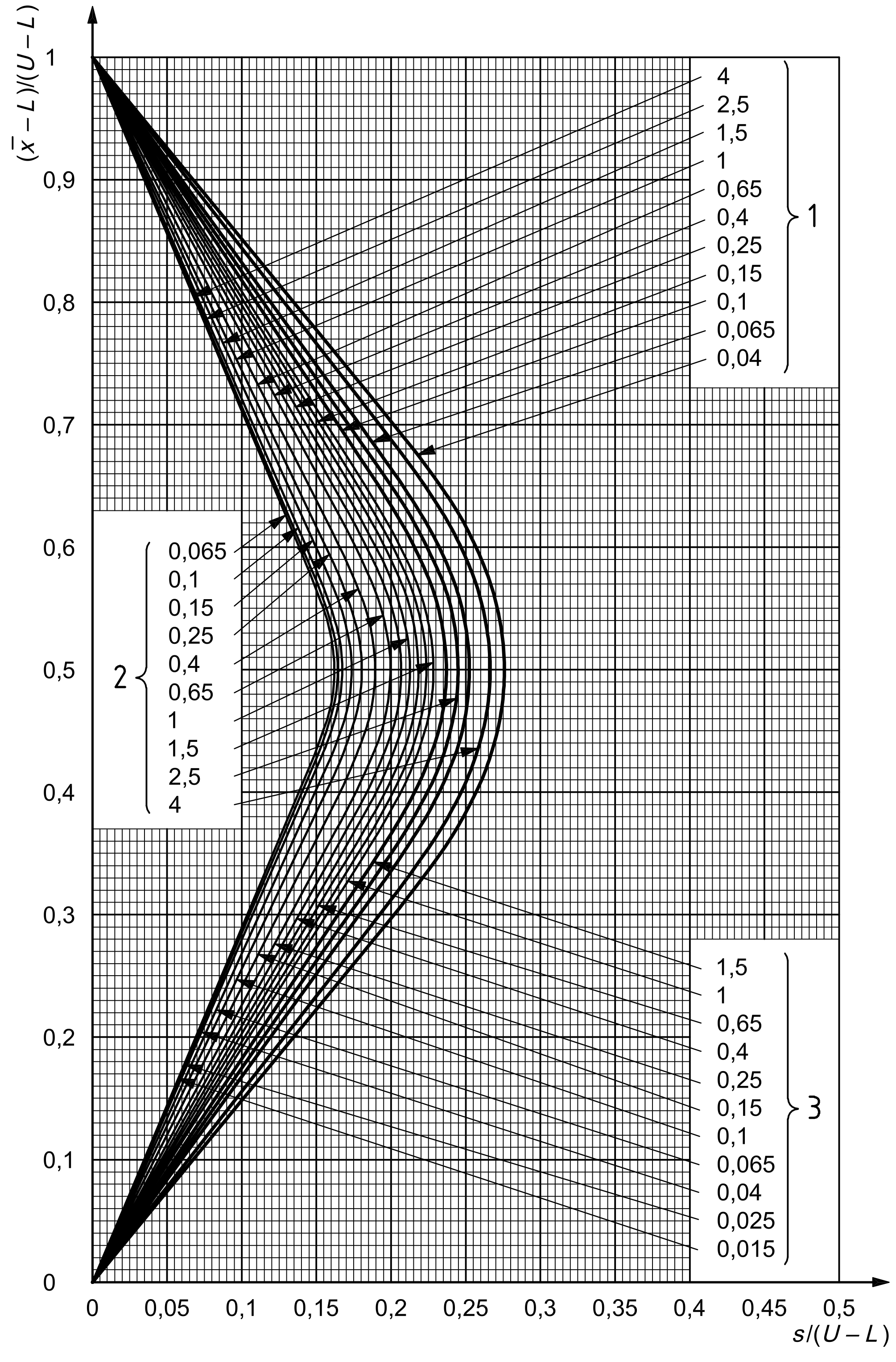
Key

1 AQL % (normal, L)

2 AQL % (tightened, L)

3 AQL % (reduced, N)

Figure 27 — Chart *s*-L — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter L (sample size 70) (sample-size code letter N for reduced inspection)



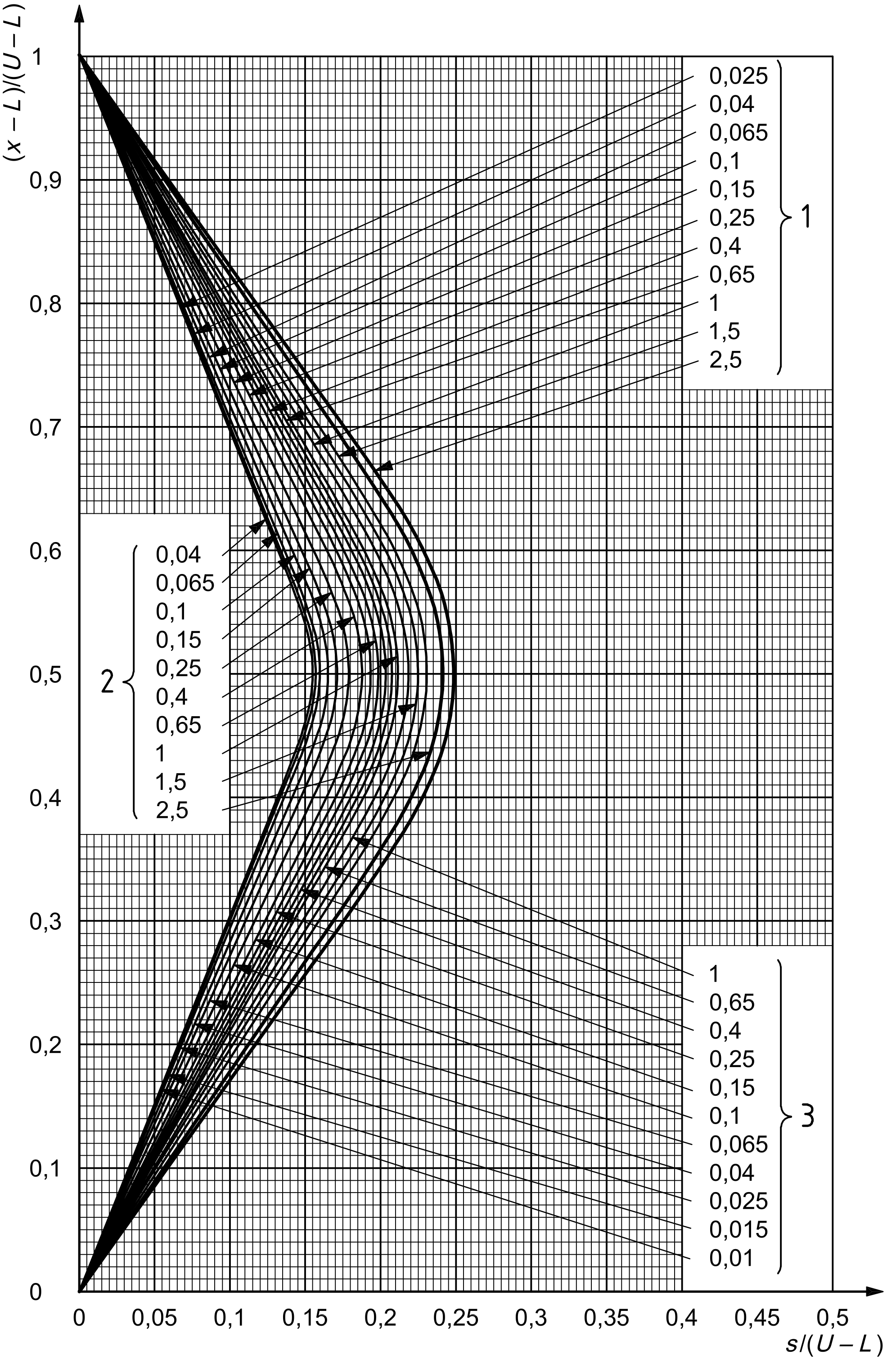
Key

1 AQL % (normal, M)

2 AQL % (tightened, M)

3 AQL % (reduced, P)

Figure 28 — Chart *s*-M — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter M (sample size 95) (sample-size code letter P for reduced inspection)



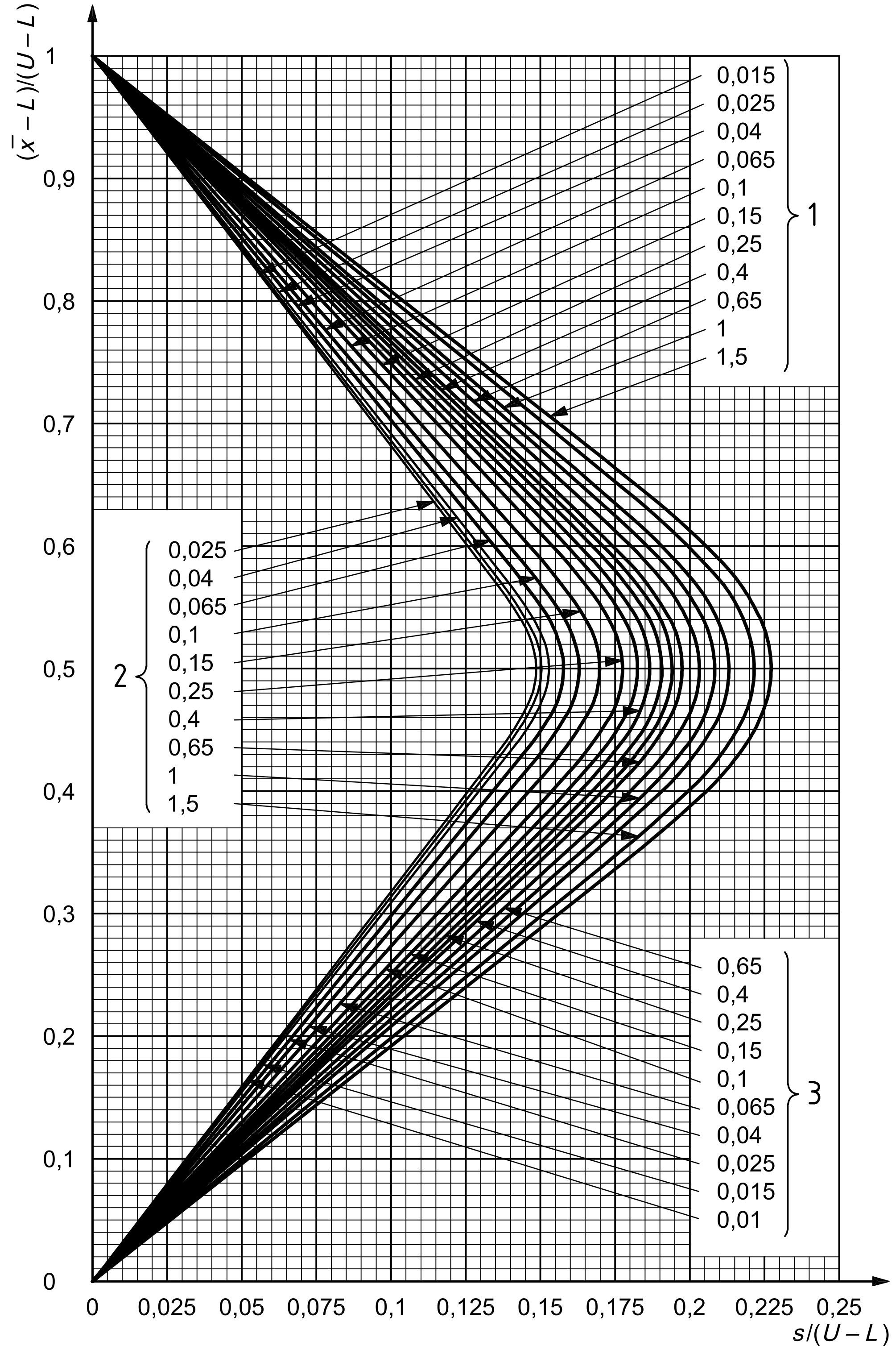
Key

1 AQL % (normal, N)

2 AQL % (tightened, N)

3 AQL % (reduced, Q)

Figure 29 — Chart *s*-N — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter N (sample size 125) (sample-size code letter Q for reduced inspection)



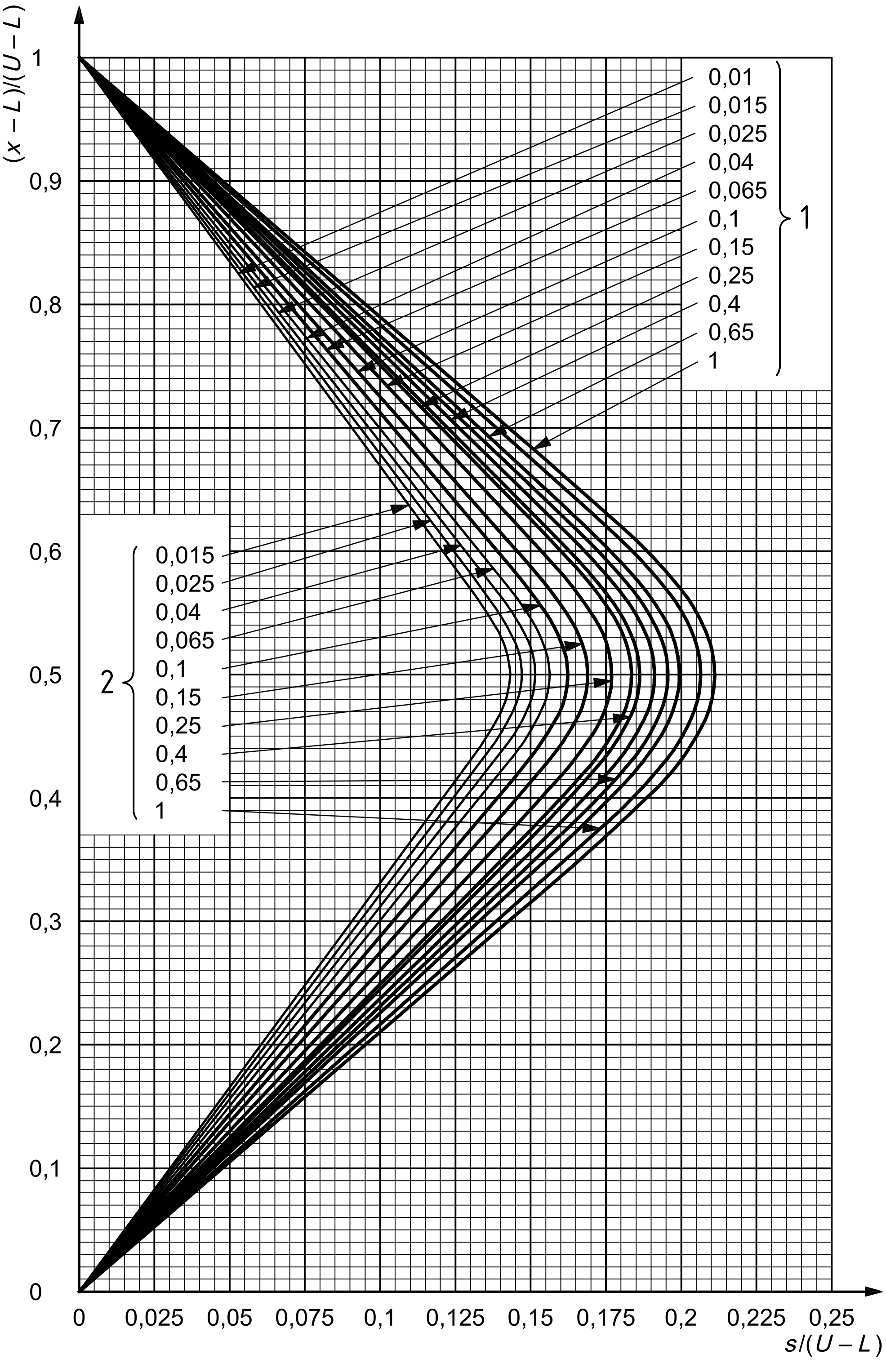
Key

1 AQL % (normal, P)

2 AQL % (tightened, P)

3 AQL % (reduced, R)

Figure 30 — Chart *s*-P — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter P (sample size 160) (sample-size code letter R for reduced inspection)

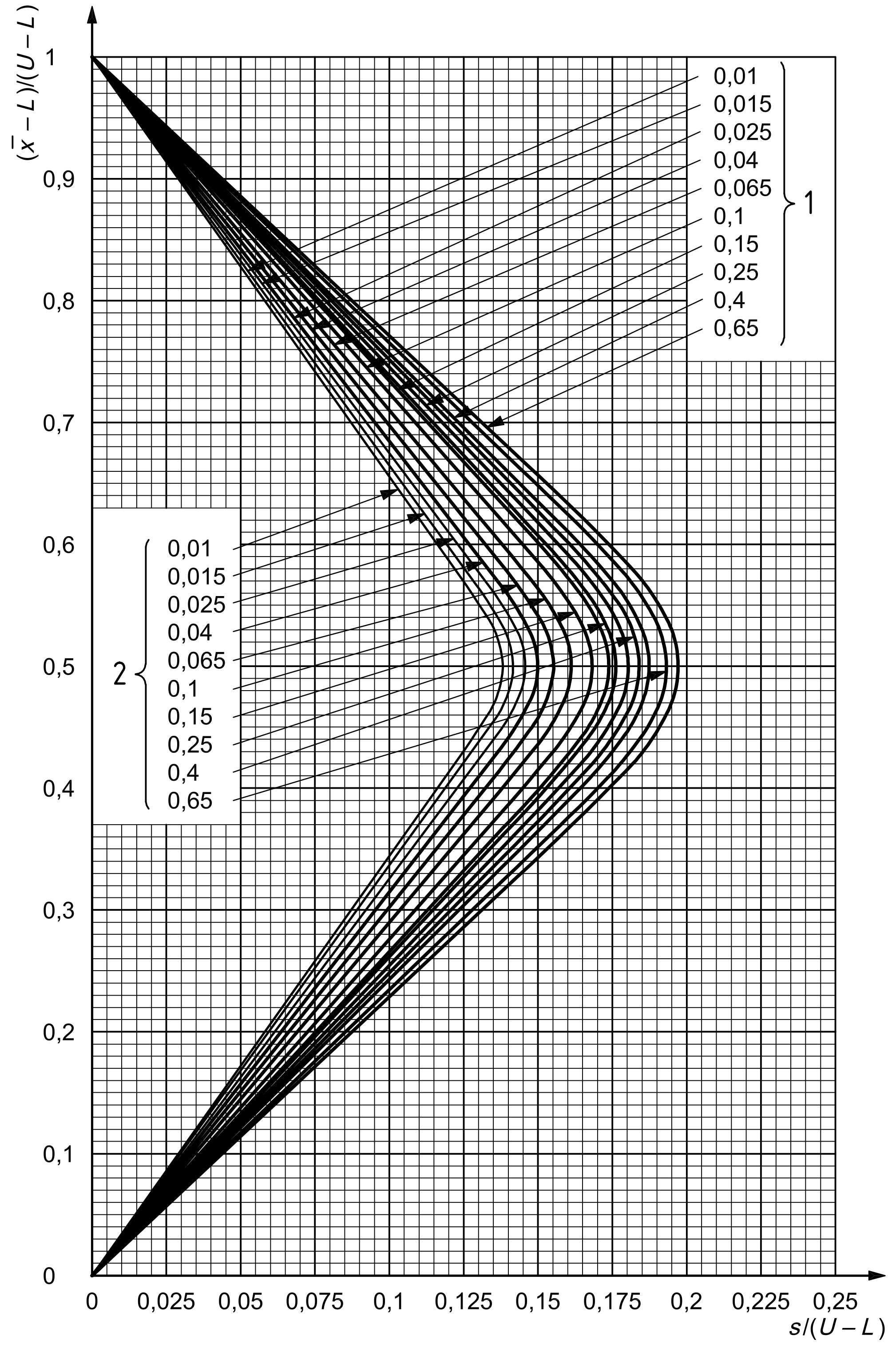


Key

1 AQL % (normal, Q)

2 AQL % (tightened, Q)

Figure 31 — Chart *s*-Q — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter Q (sample size 200)



Key

1 AQL % (normal, R)

2 AQL % (tightened, R)

Figure 32 — Chart *s*-R — Acceptance curves for double specification limits with a combined AQL requirement: "*s*" method — Sample-size code letter R (sample size 250)

Annex A
(normative)

Tables for determining the appropriate sample size

Tables A.1 and A.2 specify appropriate sample sizes.

Table A.1 — Sample-size code letters and inspection levels

Lot or batch size	Special inspection levels				General inspection levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	B	B	B	B	B	B	B
9 to 15	B	B	B	B	B	B	C
16 to 25	B	B	B	B	B	C	D
26 to 50	B	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	F	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1 200	C	C	E	F	G	J	K
1 201 to 3 200	C	D	E	G	H	K	L
3 201 to 10 000	C	D	F	G	J	L	M
10 001 to 35 000	C	D	F	H	K	M	N
35 001 to 150 000	D	E	G	J	L	N	P
150 001 to 500 000	D	E	G	J	M	P	Q
500 000 and over	D	E	H	K	N	Q	R

NOTE The sample-size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1.

Table A.2 — Sample sizes for sample-size code letters and inspection method

Sample-size code letter	"s" method		"σ" method		Equivalent attributes sample size in ISO 2859-1	
	Normal and tightened inspection	Reduced inspection	Normal and tightened inspection	Reduced inspection	Normal and tightened inspection	Reduced inspection
B	3	3	2	2	3	2
C	4	3	3	2	5	2
D	6	3	4	2	8	3
E	9	4	6	3	13	5
F	13	6	8	4	20	8
G	18	9	10	6	32	13
H	25	13	12	8	50	20
J	35	18	15	10	80	32
K	50	25	18	12	125	50
L	70	35	21	15	200	80
M	95	50	25	18	315	125
N	125	70	32	21	500	200
P	160	95	40	25	800	315
Q	200	125	50	32	1250	500
R	250	160	65	40	2000	800

NOTE The sample-size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1.

Annex B
(normative)

Form k single sampling plans for the "s" method

Tables B.1 to B.3 provide single sampling plans for the "s" method.

Table B.1 — Single sampling plans for normal inspection (master table): "*s*" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B	3													↓	0,954	0,818	0,526
C	4												↓	1,163	1,046	0,853	0,580
D	6											↓	1,395	1,275	1,108	0,902	0,587
E	9										↓	1,615	1,494	1,338	1,159	0,907	0,597
F	13									↓	1,830	1,712	1,565	1,405	1,189	0,938	0,614
G	18								↓	2,025	1,910	1,770	1,622	1,429	1,212	0,944	0,718
H	25							↓	2,215	2,102	1,969	1,829	1,652	1,457	1,225	1,035	0,809
J	35						↓	2,399	2,289	2,160	2,028	1,862	1,684	1,476	1,311	1,118	0,912
K	50					↓	2,569	2,461	2,336	2,209	2,052	1,885	1,693	1,543	1,372	1,193	0,947
L	70				↓	2,736	2,631	2,510	2,389	2,239	2,082	1,904	1,766	1,611	1,451	1,238	↑
M	95			↓	2,889	2,787	2,670	2,553	2,410	2,261	2,093	1,965	1,822	1,676	1,484	↑	
N	125		↓	3,037	2,937	2,824	2,711	2,574	2,432	2,274	2,154	2,021	1,886	1,710	↑		
P	160	↓	3,179	3,082	2,973	2,865	2,733	2,597	2,447	2,334	2,209	2,083	1,921	↑			
Q	200	3,310	3,215	3,109	3,004	2,877	2,747	2,603	2,495	2,377	2,258	2,106	↑				
R	250	3,350	3,247	3,146	3,023	2,898	2,760	2,657	2,545	2,432	2,289	↑					

NOTE 1 The sample-size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1.

NOTE 2 Symbols: ↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.2 — Single sampling plans for tightened inspection (master table): "*s*" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,954	0,818
C	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,163	1,046	0,853
D	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,395	1,275	1,108	0,902
E	9	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,615	1,494	1,338	1,159	0,907
F	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,830	1,712	1,565	1,405	1,189	0,938
G	18	↓	↓	↓	↓	↓	↓	↓	↓	↓	2,025	1,910	1,770	1,622	1,429	1,212	0,944
H	25	↓	↓	↓	↓	↓	↓	↓	↓	2,215	2,102	1,969	1,829	1,652	1,457	1,225	0,995
J	35	↓	↓	↓	↓	↓	↓	↓	2,399	2,289	2,160	2,028	1,862	1,684	1,476	1,242	1,010
K	50	↓	↓	↓	↓	↓	↓	2,569	2,461	2,336	2,209	2,052	1,885	1,693	1,481	1,277	1,044
L	70	↓	↓	↓	↓	↓	2,736	2,631	2,510	2,389	2,239	2,082	1,904	1,710	1,526	1,322	↑
M	95	↓	↓	↓	↓	2,889	2,787	2,670	2,553	2,410	2,261	2,093	1,913	1,745	1,559	↑	↑
N	125	↓	↓	↓	3,037	2,937	2,824	2,711	2,574	2,432	2,274	2,105	1,949	1,779	↑	↑	↑
P	160	↓	↓	3,179	3,082	2,973	2,865	2,733	2,597	2,447	2,288	2,141	1,984	↑	↑	↑	↑
Q	200	↓	3,310	3,215	3,109	3,004	2,877	2,747	2,603	2,452	2,313	2,165	↑	↑	↑	↑	↑
R	250	3,442	3,350	3,247	3,146	3,023	2,898	2,760	2,616	2,485	2,345	↑	↑	↑	↑	↑	↑

NOTE 1 The sample-size code letters in this International Standard correspond to those given in ISO 2859-1.

NOTE 2 Symbols:

↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.3 — Single sampling plans for reduced inspection (master table): “*s*” method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B - D	3											↓	0,954	0,907	0,818	0,526	0,023
E	4										↓	1,163	1,119	1,046	0,853	0,580	0,099
F	6									↓	1,395	1,348	1,275	1,108	0,902	0,587	0,161
G	9								↓	1,615	1,566	1,494	1,338	1,159	0,907	0,597	0,368
H	13							↓	1,830	1,782	1,712	1,565	1,405	1,189	0,938	0,763	0,461
J	18						↓	2,025	1,978	1,910	1,770	1,622	1,429	1,212	1,065	0,823	0,619
K	25					↓	2,215	2,168	2,102	1,969	1,829	1,652	1,457	1,329	1,123	0,955	0,809
L	35				↓	2,399	2,353	2,289	2,160	2,028	1,862	1,684	1,569	1,387	1,242	1,118	↑
M	50			↓	2,569	2,524	2,461	2,336	2,209	2,052	1,885	1,778	1,612	1,481	1,372	↑	
N	70		↓	2,736	2,692	2,631	2,510	2,389	2,239	2,082	1,982	1,829	1,710	1,611	↑		
P	95	↓	2,889	2,846	2,787	2,670	2,553	2,410	2,261	2,167	2,023	1,913	1,822	↑			
Q	125	3,037	2,995	2,937	2,824	2,711	2,574	2,432	2,344	2,208	2,105	2,021	↑				
R	160	3,139	3,082	2,973	2,865	2,733	2,597	2,513	2,385	2,288	2,209	↑					

NOTE 1 The sample-size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1.

NOTE 2 Symbols:
↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.
↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex C
(normative)

Form k single sampling plans for the “σ” method

Tables C.1 to C.3 provide single sampling plans for the “σ” method.

Table C.1 — Single sampling plans for normal inspection (master table): "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,620	0,478	0,273
C	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,991	0,841	0,643	0,412
D	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,296	1,148	0,964	0,760	0,478
E	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,578	1,432	1,256	1,068	0,818	0,528
F	8	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,821	1,682	1,517	1,344	1,121	0,872	0,564
G	10	↓	↓	↓	↓	↓	↓	↓	↓	2,030	1,897	1,742	1,581	1,378	1,157	0,893	0,675
H	12	↓	↓	↓	↓	↓	↓	↓	2,223	2,096	1,949	1,800	1,613	1,412	1,179	0,991	0,771
J	15	↓	↓	↓	↓	↓	↓	2,410	2,289	2,150	2,009	1,835	1,650	1,439	1,273	1,082	0,879
K	18	↓	↓	↓	↓	↓	2,576	2,459	2,327	2,193	2,029	1,857	1,662	1,511	1,340	1,162	0,919
L	21	↓	↓	↓	↓	2,738	2,627	2,500	2,374	2,218	2,057	1,876	1,737	1,582	1,422	1,210	↑
M	25	↓	↓	↓	2,890	2,783	2,661	2,540	2,393	2,240	2,070	1,941	1,797	1,650	1,459	↑	↑
N	32	↓	↓	3,041	2,937	2,820	2,704	2,563	2,419	2,258	2,136	2,001	1,866	1,690	↑	↑	↑
P	40	↓	3,186	3,086	2,974	2,862	2,727	2,589	2,436	2,321	2,194	2,068	1,905	↑	↑	↑	↑
Q	50	3,319	3,222	3,113	3,005	2,875	2,742	2,596	2,487	2,367	2,247	2,094	↑	↑	↑	↑	↑
R	65	3,359	3,254	3,150	3,025	2,897	2,758	2,653	2,539	2,426	2,281	↑	↑	↑	↑	↑	↑

NOTE 1 The sample-size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1.

NOTE 2 Symbols: ↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.2 — Single sampling plans for tightened inspection (master table): "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	2														↓	0,620	0,478
C	3													↓	0,991	0,841	0,643
D	4												↓	1,296	1,148	0,964	0,760
E	6											↓	1,578	1,432	1,256	1,068	0,818
F	8										↓	1,821	1,682	1,517	1,344	1,121	0,872
G	10									↓	2,030	1,897	1,742	1,581	1,378	1,157	0,893
H	12								↓	2,223	2,096	1,949	1,800	1,613	1,412	1,179	0,913
J	15							↓	2,410	2,289	2,150	2,009	1,835	1,650	1,439	1,204	0,976
K	18						↓	2,576	2,459	2,327	2,193	2,029	1,857	1,662	1,449	1,245	1,015
L	21					↓	2,738	2,627	2,500	2,374	2,218	2,057	1,876	1,681	1,497	1,293	↑
M	25				↓	2,890	2,783	2,661	2,540	2,393	2,240	2,070	1,888	1,719	1,534	↑	
N	32			↓	3,041	2,937	2,820	2,704	2,563	2,419	2,258	2,087	1,929	1,758	↑		
P	40		↓	3,186	3,086	2,974	2,862	2,727	2,589	2,436	2,274	2,127	1,968	↑			
Q	50	↓	3,319	3,222	3,113	3,005	2,875	2,742	2,596	2,443	2,303	2,154	↑				
R	65	3,454	3,359	3,254	3,150	3,025	2,897	2,758	2,611	2,478	2,337	↑					

NOTE 1 The sample-size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1.

NOTE 2 Symbols: ↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.3 — Single sampling plans for reduced inspection (master table): “σ” method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B - D	2											↓	0,620	0,565	0,478	0,273	0,011
E	3										↓	0,991	0,931	0,841	0,643	0,412	0,067
F	4									↓	1,296	1,236	1,148	0,964	0,760	0,478	0,129
G	6								↓	1,578	1,518	1,432	1,256	1,068	0,818	0,528	0,323
H	8							↓	1,821	1,764	1,682	1,517	1,344	1,121	0,872	0,705	0,422
J	10						↓	2,030	1,975	1,897	1,742	1,581	1,378	1,157	1,012	0,776	0,581
K	12					↓	2,223	2,170	2,096	1,949	1,800	1,613	1,412	1,283	1,078	0,913	0,771
L	15				↓	2,410	2,360	2,289	2,150	2,009	1,835	1,650	1,533	1,349	1,204	1,082	↑
M	18			↓	2,576	2,527	2,459	2,327	2,193	2,029	1,857	1,748	1,580	1,449	1,340	↑	
N	21		↓	2,738	2,691	2,627	2,500	2,374	2,218	2,057	1,956	1,801	1,681	1,582	↑		
P	25	↓	2,890	2,845	2,783	2,661	2,540	2,393	2,240	2,145	1,999	1,888	1,797	↑			
Q	32	3,041	2,998	2,937	2,820	2,704	2,563	2,419	2,328	2,191	2,087	2,001	↑				
R	40	3,144	3,086	2,974	2,862	2,727	2,589	2,503	2,373	2,274	2,194	↑					

NOTE 1 The sample-size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1.

NOTE 2 Symbols:
↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.
↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex D
(normative)

Values of f_S for maximum sample standard deviation (MSSD)

Tables D.1 to D.3 provide values of f_S for maximum sample standard deviation.

Table D.1 — Values of f_s for maximum sample standard deviation for combined control of double specification limits: normal inspection, "s" method

Code letter	Acceptance quality limit % nonconforming															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,474	0,507	0,595
C													0,376	0,393	0,425	0,481
D												0,314	0,331	0,357	0,396	0,471
E											0,274	0,289	0,310	0,338	0,386	0,464
F										0,245	0,257	0,274	0,295	0,328	0,375	0,457
G									0,224	0,234	0,248	0,264	0,289	0,321	0,372	0,426
H								0,206	0,215	0,227	0,240	0,259	0,283	0,317	0,351	0,401
J							0,192	0,200	0,209	0,220	0,235	0,254	0,279	0,302	0,335	0,376
K						0,180	0,187	0,195	0,205	0,217	0,232	0,252	0,269	0,292	0,320	0,368
L					0,170	0,176	0,183	0,191	0,202	0,214	0,230	0,243	0,261	0,281	0,312	↑
M				0,162	0,167	0,174	0,180	0,189	0,200	0,213	0,224	0,237	0,253	0,276	↑	
N			0,155	0,160	0,165	0,171	0,179	0,188	0,199	0,208	0,219	0,231	0,249	↑		
P		0,149	0,153	0,158	0,163	0,170	0,177	0,187	0,194	0,203	0,213	0,227	↑			
Q	0,143	0,147	0,152	0,156	0,162	0,169	0,177	0,183	0,191	0,199	0,211	↑				
R	0,142	0,146	0,150	0,155	0,161	0,168	0,174	0,180	0,187	0,197	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = $(U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.2 — Values of f_s for maximum sample standard deviation for combined control of double specification limits: tightened inspection, "s" method

Code letter	Acceptance quality limit % nonconforming															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B														↓	0,474	0,507
C													↓	0,376	0,393	0,425
D												↓	0,314	0,331	0,357	0,396
E											↓	0,274	0,289	0,310	0,338	0,386
F										↓	0,245	0,257	0,274	0,295	0,328	0,375
G									↓	0,224	0,234	0,248	0,264	0,289	0,321	0,372
H								↓	0,206	0,215	0,227	0,240	0,259	0,283	0,317	0,368
J							↓	0,192	0,200	0,209	0,220	0,235	0,254	0,279	0,313	0,355
K						↓	0,180	0,187	0,195	0,205	0,217	0,232	0,252	0,277	0,307	0,348
L					↓	0,170	0,176	0,183	0,191	0,202	0,214	0,230	0,249	0,271	0,299	↑
M				↓	0,162	0,167	0,174	0,180	0,189	0,200	0,213	0,228	0,245	0,266	↑	
N			↓	0,155	0,160	0,165	0,171	0,179	0,188	0,199	0,212	0,225	0,241	↑		
P		↓	0,149	0,153	0,158	0,163	0,170	0,177	0,187	0,197	0,208	0,222	↑			
Q	↓	0,143	0,147	0,152	0,156	0,162	0,169	0,177	0,186	0,196	0,206	↑				
R	0,138	0,142	0,146	0,150	0,155	0,161	0,168	0,176	0,184	0,193	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = $(U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under tightened inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.3 — Values of f_s for maximum sample standard deviation for combined control of double specification limits: reduced inspection, "s" method

Code letter	Acceptance quality limit % nonconforming															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B-D											↓	0,474	0,485	0,507	0,595	0,849
E										↓	0,376	0,382	0,393	0,425	0,481	0,625
F									↓	0,314	0,320	0,331	0,357	0,396	0,471	0,623
G								↓	0,274	0,280	0,289	0,310	0,338	0,386	0,464	0,542
H							↓	0,245	0,250	0,257	0,274	0,295	0,328	0,375	0,416	0,507
J						↓	0,224	0,228	0,234	0,248	0,264	0,289	0,321	0,347	0,399	0,455
K					↓	0,206	0,210	0,215	0,227	0,240	0,259	0,283	0,301	0,335	0,368	0,401
L				↓	0,192	0,195	0,200	0,209	0,220	0,235	0,254	0,267	0,291	0,313	0,335	↑
M			↓	0,180	0,183	0,187	0,195	0,205	0,217	0,232	0,243	0,261	0,277	0,292	↑	
N		↓	0,170	0,173	0,176	0,183	0,191	0,202	0,214	0,223	0,237	0,249	0,261	↑		
P	↓	0,162	0,164	0,167	0,174	0,180	0,189	0,200	0,207	0,219	0,228	0,237	↑			
Q	0,155	0,157	0,160	0,165	0,171	0,179	0,188	0,194	0,203	0,212	0,219	↑				
R	0,150	0,153	0,158	0,163	0,170	0,177	0,183	0,191	0,197	0,203	↑					

NOTE The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = $(U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under reduced inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD then there is a possibility, but not a certainty, that the lot will be accepted.

Annex E
(normative)

Values of f_σ for maximum process standard deviation (MPSD)

Table E.1 provides values for maximum process standard deviation. The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for combined control of double specification limits when the process variability is known. If the process standard deviation is less than the MPSD, then there is a possibility, but not a certainty, that the lot will be accepted.

The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit U and the lower specification limit L, i.e. MPSD = $(U - L)f_\sigma$.

Table E.1 — Values of f_σ for maximum process standard deviation for combined control of double specification limits: "σ" method

Acceptance quality limit % nonconforming	f_σ
0,010	0,125
0,015	0,129
0,025	0,132
0,040	0,137
0,065	0,141
0,10	0,147
0,15	0,152
0,25	0,157
0,40	0,165
0,65	0,174
1,0	0,184
1,5	0,194
2,5	0,206
4,0	0,223
6,5	0,243
10,0	0,271

Annex F
(normative)

Estimating the process fraction nonconforming for sample size 3: "s" method

Table F.1 provides values for the estimated process fraction nonconforming.

Table F.1 — Estimated process fraction nonconforming, $\hat{p}$, as a function of the quality statistic Q

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,00	0,5000	0,4997	0,4994	0,4990	0,4987	0,4984	0,4981	0,4978	0,4975	0,4971
0,01	0,4968	0,4965	0,4962	0,4959	0,4955	0,4952	0,4949	0,4946	0,4943	0,4940
0,02	0,4936	0,4933	0,4930	0,4927	0,4924	0,4920	0,4917	0,4914	0,4911	0,4908
0,03	0,4904	0,4901	0,4898	0,4895	0,4892	0,4889	0,4885	0,4882	0,4879	0,4876
0,04	0,4873	0,4869	0,4866	0,4863	0,4860	0,4857	0,4854	0,4850	0,4847	0,4844
0,05	0,4841	0,4838	0,4834	0,4831	0,4828	0,4825	0,4822	0,4818	0,4815	0,4812
0,06	0,4809	0,4806	0,4803	0,4799	0,4796	0,4793	0,4790	0,4787	0,4783	0,4780
0,07	0,4777	0,4774	0,4771	0,4767	0,4764	0,4761	0,4758	0,4755	0,4751	0,4748
0,08	0,4745	0,4742	0,4739	0,4735	0,4732	0,4729	0,4726	0,4723	0,4720	0,4716
0,09	0,4713	0,4710	0,4707	0,4704	0,4700	0,4697	0,4694	0,4691	0,4688	0,4684
0,10	0,4681	0,4678	0,4675	0,4672	0,4668	0,4665	0,4662	0,4659	0,4656	0,4652
0,11	0,4649	0,4646	0,4643	0,4640	0,4636	0,4633	0,4630	0,4627	0,4624	0,4620
0,12	0,4617	0,4614	0,4611	0,4607	0,4604	0,4601	0,4598	0,4595	0,4591	0,4588
0,13	0,4585	0,4582	0,4579	0,4575	0,4572	0,4569	0,4566	0,4563	0,4559	0,4556
0,14	0,4553	0,4550	0,4546	0,4543	0,4540	0,4537	0,4534	0,4530	0,4527	0,4524
0,15	0,4521	0,4518	0,4514	0,4511	0,4508	0,4505	0,4501	0,4498	0,4495	0,4492
0,16	0,4489	0,4485	0,4482	0,4479	0,4476	0,4472	0,4469	0,4466	0,4463	0,4459
0,17	0,4456	0,4453	0,4450	0,4447	0,4443	0,4440	0,4437	0,4434	0,4430	0,4427
0,18	0,4424	0,4421	0,4417	0,4414	0,4411	0,4408	0,4404	0,4401	0,4398	0,4395
0,19	0,4392	0,4388	0,4385	0,4382	0,4379	0,4375	0,4372	0,4369	0,4366	0,4362
0,20	0,4359	0,4356	0,4353	0,4349	0,4346	0,4343	0,4340	0,4336	0,4333	0,4330
0,21	0,4327	0,4323	0,4320	0,4317	0,4314	0,4310	0,4307	0,4304	0,4300	0,4297
0,22	0,4294	0,4291	0,4287	0,4284	0,4281	0,4278	0,4274	0,4271	0,4268	0,4265
0,23	0,4261	0,4258	0,4255	0,4251	0,4248	0,4245	0,4242	0,4238	0,4235	0,4232
0,24	0,4229	0,4225	0,4222	0,4219	0,4215	0,4212	0,4209	0,4206	0,4202	0,4199
0,25	0,4196	0,4192	0,4189	0,4186	0,4183	0,4179	0,4176	0,4173	0,4169	0,4166
0,26	0,4163	0,4159	0,4156	0,4153	0,4150	0,4146	0,4143	0,4140	0,4136	0,4133
0,27	0,4130	0,4126	0,4123	0,4120	0,4117	0,4113	0,4110	0,4107	0,4103	0,4100
0,28	0,4097	0,4093	0,4090	0,4087	0,4083	0,4080	0,4077	0,4073	0,4070	0,4067
0,29	0,4063	0,4060	0,4057	0,4053	0,4050	0,4047	0,4043	0,4040	0,4037	0,4033
0,30	0,4030	0,4027	0,4023	0,4020	0,4017	0,4013	0,4010	0,4007	0,4003	0,4000

Table F.1 *(continued)*

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,31	0,3997	0,3993	0,3990	0,3987	0,3983	0,3980	0,3977	0,3973	0,3970	0,3967
0,32	0,3963	0,3960	0,3956	0,3953	0,3950	0,3946	0,3943	0,3940	0,3936	0,3933
0,33	0,3930	0,3926	0,3923	0,3919	0,3916	0,3913	0,3909	0,3906	0,3902	0,3899
0,34	0,3896	0,3892	0,3889	0,3886	0,3882	0,3879	0,3875	0,3872	0,3869	0,3865
0,35	0,3862	0,3858	0,3855	0,3852	0,3848	0,3845	0,3841	0,3838	0,3835	0,3831
0,36	0,3828	0,3824	0,3821	0,3818	0,3814	0,3811	0,3807	0,3804	0,3800	0,3797
0,37	0,3794	0,3790	0,3787	0,3783	0,3780	0,3776	0,3773	0,3770	0,3766	0,3763
0,38	0,3759	0,3756	0,3752	0,3749	0,3745	0,3742	0,3739	0,3735	0,3732	0,3728
0,39	0,3725	0,3721	0,3718	0,3714	0,3711	0,3707	0,3704	0,3701	0,3697	0,3694
0,40	0,3690	0,3687	0,3683	0,368	0,3676	0,3673	0,3669	0,3666	0,3662	0,3659
0,41	0,3655	0,3652	0,3648	0,3645	0,3641	0,3638	0,3634	0,3631	0,3627	0,3624
0,42	0,3620	0,3617	0,3613	0,3610	0,3606	0,3603	0,3599	0,3596	0,3592	0,3589
0,43	0,3585	0,3582	0,3578	0,3575	0,3571	0,3567	0,3564	0,356	0,3557	0,3553
0,44	0,3550	0,3546	0,3543	0,3539	0,3536	0,3532	0,3528	0,3525	0,3521	0,3518
0,45	0,3514	0,3511	0,3507	0,3504	0,3500	0,3496	0,3493	0,3489	0,3486	0,3482
0,46	0,3478	0,3475	0,3471	0,3468	0,3464	0,3461	0,3457	0,3453	0,3450	0,3446
0,47	0,3443	0,3439	0,3435	0,3432	0,3428	0,3424	0,3421	0,3417	0,3414	0,3410
0,48	0,3406	0,3403	0,3399	0,3395	0,3392	0,3388	0,3385	0,3381	0,3377	0,3374
0,49	0,3370	0,3366	0,3363	0,3359	0,3355	0,3352	0,3348	0,3344	0,3341	0,3337
0,50	0,3333	0,3330	0,3326	0,3322	0,3319	0,3315	0,3311	0,3308	0,3304	0,3300
0,51	0,3296	0,3293	0,3289	0,3285	0,3282	0,3278	0,3274	0,3270	0,3267	0,3263
0,52	0,3259	0,3256	0,3252	0,3248	0,3244	0,3241	0,3237	0,3233	0,3229	0,3226
0,53	0,3222	0,3218	0,3214	0,3211	0,3207	0,3203	0,3199	0,3196	0,3192	0,3188
0,54	0,3184	0,3180	0,3177	0,3173	0,3169	0,3165	0,3161	0,3158	0,3154	0,3150
0,55	0,3146	0,3142	0,3139	0,3135	0,3131	0,3127	0,3123	0,3120	0,3116	0,3112
0,56	0,3108	0,3104	0,3100	0,3096	0,3093	0,3089	0,3085	0,3081	0,3077	0,3073
0,57	0,3069	0,3066	0,3062	0,3058	0,3054	0,3050	0,3046	0,3042	0,3038	0,3034
0,58	0,3031	0,3027	0,3023	0,3019	0,3015	0,3011	0,3007	0,3003	0,2999	0,2995
0,59	0,2991	0,2987	0,2983	0,2979	0,2975	0,2972	0,2968	0,2964	0,2960	0,2956
0,60	0,2952	0,2948	0,2944	0,2940	0,2936	0,2932	0,2928	0,2924	0,2920	0,2916
0,61	0,2912	0,2908	0,2904	0,2900	0,2896	0,2892	0,2888	0,2883	0,2879	0,2875
0,62	0,2871	0,2867	0,2863	0,2859	0,2855	0,2851	0,2847	0,2843	0,2839	0,2835
0,63	0,2831	0,2826	0,2822	0,2818	0,2814	0,2810	0,2806	0,2802	0,2798	0,2793
0,64	0,2789	0,2785	0,2781	0,2777	0,2773	0,2769	0,2764	0,2760	0,2756	0,2752
0,65	0,2748	0,2743	0,2739	0,2735	0,2731	0,2727	0,2722	0,2718	0,2714	0,2710
0,66	0,2706	0,2701	0,2697	0,2693	0,2689	0,2684	0,2680	0,2676	0,2672	0,2667
0,67	0,2663	0,2659	0,2654	0,2650	0,2646	0,2641	0,2637	0,2633	0,2628	0,2624
0,68	0,2620	0,2615	0,2611	0,2607	0,2602	0,2598	0,2594	0,2589	0,2585	0,2580
0,69	0,2576	0,2572	0,2567	0,2563	0,2558	0,2554	0,2550	0,2545	0,2541	0,2536
0,70	0,2532	0,2527	0,2523	0,2518	0,2514	0,2509	0,2505	0,2500	0,2496	0,2491

Table F.1 *(continued)*

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,71	0,2487	0,2482	0,2478	0,2473	0,2469	0,2464	0,2460	0,2455	0,2451	0,2446
0,72	0,2441	0,2437	0,2432	0,2428	0,2423	0,2418	0,2414	0,2409	0,2405	0,2400
0,73	0,2395	0,2391	0,2386	0,2381	0,2377	0,2372	0,2367	0,2362	0,2358	0,2353
0,74	0,2348	0,2344	0,2339	0,2334	0,2329	0,2324	0,2320	0,2315	0,2310	0,2305
0,75	0,2301	0,2296	0,2291	0,2286	0,2281	0,2276	0,2272	0,2267	0,2262	0,2257
0,76	0,2252	0,2247	0,2242	0,2237	0,2232	0,2227	0,2222	0,2217	0,2213	0,2208
0,77	0,2203	0,2198	0,2193	0,2188	0,2183	0,2177	0,2172	0,2167	0,2162	0,2157
0,78	0,2152	0,2147	0,2142	0,2137	0,2132	0,2127	0,2121	0,2116	0,2111	0,2106
0,79	0,2101	0,2096	0,2090	0,2085	0,2080	0,2075	0,2069	0,2064	0,2059	0,2054
0,80	0,2048	0,2043	0,2038	0,2032	0,2027	0,2022	0,2016	0,2011	0,2006	0,2000
0,81	0,1995	0,1989	0,1984	0,1978	0,1973	0,1967	0,1962	0,1956	0,1951	0,1945
0,82	0,1940	0,1934	0,1929	0,1923	0,1917	0,1912	0,1906	0,1900	0,1895	0,1889
0,83	0,1883	0,1878	0,1872	0,1866	0,1860	0,1855	0,1849	0,1843	0,1837	0,1831
0,84	0,1826	0,1820	0,1814	0,1808	0,1802	0,1796	0,1790	0,1784	0,1778	0,1772
0,85	0,1766	0,1760	0,1754	0,1748	0,1742	0,1736	0,1729	0,1723	0,1717	0,1711
0,86	0,1705	0,1698	0,1692	0,1686	0,1680	0,1673	0,1667	0,1660	0,1654	0,1648
0,87	0,1641	0,1635	0,1628	0,1622	0,1615	0,1609	0,1602	0,1595	0,1589	0,1582
0,88	0,1575	0,1569	0,1562	0,1555	0,1548	0,1542	0,1535	0,1528	0,1521	0,1514
0,89	0,1507	0,1500	0,1493	0,1486	0,1479	0,1472	0,1465	0,1457	0,1450	0,1443
0,90	0,1436	0,1428	0,1421	0,1414	0,1406	0,1399	0,1391	0,1384	0,1376	0,1368
0,91	0,1361	0,1353	0,1345	0,1338	0,1330	0,1322	0,1314	0,1306	0,1298	0,1290
0,92	0,1282	0,1274	0,1266	0,1257	0,1249	0,1241	0,1232	0,1224	0,1215	0,1207
0,93	0,1198	0,1189	0,1181	0,1172	0,1163	0,1154	0,1145	0,1136	0,1127	0,1118
0,94	0,1108	0,1099	0,1089	0,1080	0,1070	0,1061	0,1051	0,1041	0,1031	0,1021
0,95	0,1011	0,1001	0,0990	0,0980	0,0969	0,0959	0,0948	0,0937	0,0926	0,0915
0,96	0,0903	0,0892	0,0880	0,0869	0,0857	0,0845	0,0832	0,0820	0,0807	0,0795
0,97	0,0782	0,0768	0,0755	0,0741	0,0727	0,0713	0,0699	0,0684	0,0669	0,0653
0,98	0,0638	0,0621	0,0605	0,0588	0,0570	0,0552	0,0533	0,0514	0,0494	0,0473
0,99	0,0451	0,0427	0,0403	0,0377	0,0349	0,0318	0,0285	0,0247	0,0201	0,0142
1,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

NOTE For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Annex G
(normative)

Type p^* single sampling plans

Table G.1 maximum allowable value for type p^* single sampling plans.

Table G.1 — Maximum allowable values, p^* , of the estimated process fraction nonconforming for sample sizes 3 and 4: "s" method

Inspection severity	Sample size n	Acceptance quality limit % nonconforming					
		1,0	1,5	2,5	4,0	6,5	10,0
		p^*	p^*	p^*	p^*	p^*	p^*
Tightened	3					0,1905	0,2494
	4				0,1123	0,1513	0,2157
Normal	3				0,1905	0,2494	0,3495
	4			0,1123	0,1513	0,2157	0,3067
Reduced	3		0,1905	0,2124	0,2494	0,3495	0,4937
	4	0,1123	0,1270	0,1513	0,2157	0,3067	0,4670

Annex H
(normative)

Values of c_u for upper control limit on the sample standard deviation

Table H.1 provides values of c_u for upper limits on sample standard deviation.

Table H.1 — Values of c_u for upper control limit on the sample standard deviation

Sample size n	Factor c_u	Sample size n	Factor c_u	Sample size n	Factor c_u	Sample size n	Factor c_u
2	2,800	10	1,617	25	1,377	70	1,221
3	2,297	12	1,558	32	1,331	95	1,189
4	2,065	13	1,534	35	1,316	125	1,165
6	1,827	15	1,494	40	1,295	160	1,145
8	1,700	18	1,448	50	1,263	200	1,130
9	1,654	21	1,413	65	1,230	250	1,116

Annex I
(normative)

Supplementary acceptability constants for qualifying towards reduced inspection

Table I.1 provides supplementary acceptability constants.

Table I.1 — Supplementary acceptability constants for qualifying towards reduced inspection

Sample size code letter	**AQL** %	**Acceptability constant for AQL that is one step tighter**	
		"s" method	"σ" method
		k	k
B	4,0	1,118	0,829
C	2,5	1,325	1,201
D	1,5	1,516	1,452
E	1,0	1,740	1,735
F	0,65	1,967	1,989
G	0,40	2,153	2,185
H	0,25	2,350	2,384
J	0,15	2,503	2,532
K	0,10	2,678	2,702
L	0,065	2,856	2,875
M	0,040	3,002	3,018
N	0,025	3,157	3,176
P	0,015	3,272	3,290
Q	0,01	3,407	3,426
R	0,01	3,448	3,466

Annex J
(normative)

Procedures for obtaining s and σ

J.1 Procedure for obtaining s

J.1.1 The estimate from a sample of the standard deviation of a population is generally denoted by the symbol s. Its value may be obtained from the mathematical formula

$$s = \sqrt{\frac{\sum_{j=1}^{n}(x_j - \bar{x})^2}{n-1}} \tag{J.1}$$

where x_j is the value of the quality characteristic of the jth item in a sample of n articles, expressed as a decimal fraction, and $\bar{x}$ is the mean value of the x_j, i.e.

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j \tag{J.2}$$

J.1.2 The above formula for s is not recommended for the purpose of computation, as it tends to introduce an unnecessary amount of rounding error. An equivalent but computationally better formula is

$$s = \sqrt{\frac{n\sum_{j=1}^{n} x_j^2 - (\sum_{j=1}^{n} x_j)^2}{n(n-1)}} \tag{J.3}$$

J.1.3 If the variability is very small relative to the mean, i.e. s is very small in comparison with $\bar{x}$, this formula can be improved upon still further by subtracting a suitable arbitrary constant a from all the values of before computing s, i.e.

$$s = \sqrt{\frac{n\sum_{j=1}^{n}(x_j - a)^2 - \{\sum_{j=1}^{n}(x_j - a)\}^2}{n(n-1)}} \tag{J.4}$$

J.1.4 Many pocket calculators have a standard deviation function key. Unfortunately, sometimes the sample size n is used by the machine in the denominator instead of $n - 1$. If it is planned to use a calculator function, or a computer program, it is important to check that the formula used by the machine is equivalent to Equation (J.1). A simple check is to find the standard deviation of the three numbers 0, 1 and 2. The sample size n is 3, the sample mean is 1, the deviations from the mean are – 1, 0 and 1, the squares of the deviations are 1, 0 and 1, the sum of squares of the deviations is 2, so from Equation (J.1) we have

$$s = \sqrt{\frac{2}{2}} = \sqrt{1} = 1.$$

If the computer or calculator is erroneously using n instead of $n - 1$ in the denominator, then the result of the calculation will be

$$s = \sqrt{\frac{2}{3}} = 0{,}8165.$$

Use of n in the denominator shall be avoided, for otherwise the acceptance criterion is weakened and the AOQL protection to the consumer is lost.

NOTE It is instructive to work through the use of Equation (J.3) for this example. It is found that

$$s = \sqrt{\frac{3 \times (0^2 + 1^2 + 2^2) - (0 + 1 + 2)^2}{3 \times (3 - 1)}} = \sqrt{\frac{3 \times (0 + 1 + 4) - 3^2}{3 \times 2}} = \sqrt{\frac{3 \times 5 - 9}{6}} = \sqrt{\frac{6}{6}} = 1,$$

as before.

J.2 Procedure for obtaining σ

J.2.1 If it appears from the control chart that the value of s is in control, σ may be presumed to be the weighted root mean square of s given by the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m} (n_i - 1) s_i^2}{\sum_{i=1}^{m} (n_i - 1)}}$$

where

m is the number of lots;

n_i is the sample size from the ith lot;

s_i is the sample standard deviation from the ith lot.

J.2.2 If the sample sizes from each of the lots are equal, then the above formula simplifies to

$$\sigma = \sqrt{\frac{\sum_{i=1}^{m} s_i^2}{m}}.$$

Annex K
(informative)

Consumer's risk qualities

K.1 For a given sampling plan, the consumer's risk quality is the process quality for which the probability of accepting a given lot is 10 %.

K.2 For the "s" method, the consumer's risk quality is the solution in p to the equation $F_{n-1,\sqrt{n}k_p}(\sqrt{n}k) = 0{,}90$ where n is the sample size, k is the "s" method acceptability constant, k_p is the $(1 - p)$ fractile of the standard normal distribution and $F_{n-1,\sqrt{n}k_p}(.)$ is the distribution function of the non-central t-distribution with $n - 1$ degrees of freedom and non-centrality parameter $\sqrt{n}k_p$.

K.3 Consumer's risk qualities for the "s" method plans of this International Standard are given in Tables K.1, K.3 and K.5 for normal, tightened and reduced inspection respectively.

K.4 For the "σ" method, the consumer's risk quality is given by the formula $\Phi\{(1{,}2816/\sqrt{n}) - k\}$ where n is the sample size, k is the "σ" method acceptability constant and $\Phi(.)$ is the distribution function of the standard normal distribution.

K.5 Consumer's risk qualities for the "σ" method plans of this part of ISO 3951 are given in Tables K.2, K.4 and K.6 for normal, tightened and reduced inspection respectively.

Table K.1 — Consumer's risk quality (in percent) for normal inspection: "*s*" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	3													↓	52,9	56,0	63,1
C	4												↓	41,3	44,2	49,1	56,6
D	6											↓	8,2	31,0	35,3	40,9	50,5
E	9										↓	18,2	20,7	24,2	28,6	35,7	45,5
F	13									↓	11,4	13,3	15,9	19,2	24,3	31,2	41,6
G	18								↓	7,15	8,51	10,4	12,8	16,4	21,3	28,6	35,7
H	25							↓	4,33	5,27	6,58	8,23	10,8	14,2	19,2	24,0	30,7
J	35						↓	2,53	3,15	4,02	5,12	6,82	9,11	12,5	15,7	20,2	25,9
K	50					↓	1,46	1,86	2,42	3,13	4,25	5,78	8,05	10,3	13,3	17,1	23,5
L	70				↓	0,836	1,08	1,43	1,88	2,59	3,58	5,05	6,49	8,49	11,0	15,2	↑
M	95			↓	0,490	0,640	0,860	1,14	1,60	2,24	3,20	4,14	5,45	7,11	9,86	↑	
N	125		↓	0,289	0,382	0,519	0,699	0,988	1,39	2,01	2,61	3,46	4,54	6,34	↑		
P	160	↓	0,172	0,229	0,314	0,425	0,606	0,862	1,25	1,64	2,18	2,87	4,03	↑			
Q	200	0,105	0,141	0,195	0,265	0,381	0,545	0,797	1,05	1,40	1,86	2,61	↑				
R	250	0,0849	0,118	0,161	0,234	0,336	0,495	0,653	0,876	1,16	1,65	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10% of lots will be expected to be accepted.

Table K.2 — Consumer's risk quality (in percent) for normal inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	2													↓	61,3	66,6	73,7
C	3												↓	40,1	46,0	53,9	62,8
D	4											↓	25,6	30,6	37,3	45,3	56,5
E	6										↓	14,6	18,2	23,2	29,3	38,4	49,8
F	8									↓	8,57	11,0	14,4	18,6	25,2	33,8	45,6
G	10								↓	5,21	6,79	9,07	12,0	16,5	22,6	31,3	39,4
H	12							↓	3,19	4,22	5,72	7,64	10,7	14,9	20,9	26,7	34,4
J	15						↓	1,88	2,51	3,45	4,67	6,63	9,36	13,4	17,3	22,6	29,2
K	18					↓	1,15	1,55	2,14	2,93	4,21	6,00	8,69	11,3	15,0	19,5	26,9
L	21				↓	0,698	0,945	1,32	1,81	2,63	3,77	5,52	7,25	9,64	12,7	17,6	↑
M	25			↓	0,422	0,576	0,809	1,12	1,63	2,36	3,49	4,60	6,17	8,17	11,5	↑	
N	32		↓	0,244	0,336	0,475	0,662	0,973	1,42	2,11	2,81	3,80	5,06	7,17	↑		
P	40	↓	0,143	0,197	0,279	0,391	0,580	0,851	1,28	1,71	2,32	3,11	4,43	↑			
Q	50	0,0851	0,118	0,169	0,237	0,353	0,522	0,787	1,06	1,44	1,94	2,79	↑				
R	65	0,0687	0,0984	0,139	0,208	0,309	0,467	0,631	0,866	1,17	1,69	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.3 — Consumer's risk quality (in percent) under tightened inspection: "*s*" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	3														↓	52,9	56,0
C	4													↓	41,3	44,2	49,1
D	6												↓	28,2	31,0	35,3	40,9
E	9											↓	18,2	20,7	24,2	28,6	35,7
F	13										↓	11,4	13,3	15,9	19,2	24,3	31,2
G	18									↓	7,15	8,51	10,4	12,8	16,4	21,3	28,6
H	25								↓	4,33	5,27	6,58	8,23	10,8	14,2	19,2	26,3
J	35							↓	2,53	3,15	4,02	5,12	6,82	9,11	12,5	17,3	23,1
K	50						↓	1,46	1,86	2,42	3,13	4,25	5,78	8,05	11,3	15,2	20,8
L	70					↓	0,836	1,08	1,43	1,88	2,59	3,58	5,05	7,17	9,76	13,4	↑
M	95				↓	0,490	0,640	0,860	1,14	1,60	2,24	3,20	4,58	6,28	8,70	↑	
N	125			↓	0,289	0,382	0,519	0,699	0,988	1,39	2,01	2,90	4,01	5,58	↑		
P	160		↓	0,172	0,229	0,314	0,425	0,606	0,862	1,25	1,82	2,54	3,54	↑			
Q	200	↓	0,105	0,141	0,195	0,265	0,381	0,545	0,797	1,17	1,63	2,29	↑				
R	250	0,0628	0,0849	0,118	0,161	0,234	0,336	0,495	0,728	1,02	1,44	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.4 — Consumer's risk quality (in percent) for tightened inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	2														↓	61,3	66,6
C	3													↓	40,1	46,0	53,9
D	4												↓	25,6	30,6	37,3	45,3
E	6											↓	14,6	18,2	23,2	29,3	38,4
F	8										↓	8,57	11,0	14,4	18,6	25,2	33,8
G	10									↓	5,21	6,79	9,07	12,0	16,5	22,6	31,3
H	12								↓	3,19	4,22	5,72	7,64	10,7	14,9	20,9	29,3
J	15							↓	1,88	2,51	3,45	4,67	6,63	9,36	13,4	19,1	25,9
K	18						↓	1,15	1,55	2,14	2,93	4,21	6,00	8,69	12,6	17,3	23,8
L	21					↓	0,698	0,945	1,32	1,81	2,63	3,77	5,52	8,06	11,2	15,5	↑
M	25				↓	0,422	0,576	0,809	1,12	1,63	2,36	3,49	5,14	7,18	10,1	↑	
N	32			↓	0,244	0,336	0,475	0,622	0,973	1,42	2,11	3,14	4,43	6,28	↑		
P	40		↓	0,143	0,197	0,279	0,391	0,580	0,851	1,28	1,92	2,72	3,88	↑			
Q	50	↓	0,0851	0,118	0,169	0,237	0,353	0,522	0,787	1,19	1,69	2,43	↑				
R	65	0,0492	0,0687	0,0984	0,139	0,208	0,309	0,467	0,710	1,02	1,47	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.5 — Consumer's risk quality (in percent) under reduced inspection: "s" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	3											↓	52,9	54,0	56,0	63,1	76,4
E	4										↓	41,3	42,4	44,2	49,1	56,6	70,9
F	6									↓	28,2	29,3	31,0	35,3	40,9	50,5	64,6
G	9								↓	18,2	19,2	20,7	24,2	28,6	35,7	45,5	53,4
H	13							↓	11,4	12,1	13,3	15,9	19,2	24,3	31,2	36,6	46,9
J	18						↓	7,15	7,68	8,51	10,4	12,8	16,4	21,3	25,2	32,3	39,0
K	25					↓	4,33	4,70	5,27	6,58	8,23	10,8	14,2	16,8	21,7	26,3	30,7
L	35				↓	2,53	2,78	3,15	4,02	5,12	6,82	9,11	10,9	14,2	17,3	20,2	↑
M	50			↓	1,46	1,62	1,86	2,42	3,13	4,25	5,78	6,98	9,20	11,3	13,3	↑	
N	70		↓	0,836	0,931	1,08	1,43	1,88	2,59	3,58	4,35	5,80	7,17	8,49	↑		
P	95	↓	0,490	0,549	0,640	0,860	1,14	1,60	2,24	2,74	3,69	4,58	5,45	↑			
Q	125	0,289	0,326	0,382	0,519	0,699	0,988	1,39	1,71	2,32	2,90	3,46	↑				
R	160	0,194	0,229	0,314	0,425	0,606	0,862	1,06	1,45	1,82	2,18	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Table K.6 — Consumer's risk quality (in percent) for reduced inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	2											↓	61,3	63,3	66,6	73,7	81,5
E	3										↓	40,1	42,4	46,0	53,9	62,8	74,9
F	4									↓	25,6	27,6	30,6	37,3	45,3	56,5	69,6
G	6								↓	14,6	16,0	18,2	23,2	29,3	38,4	49,8	57,9
H	8							↓	8,57	9,49	11,0	14,4	18,6	25,2	33,8	40,1	51,2
J	10						↓	5,21	5,82	6,79	9,07	12,0	16,5	22,6	27,2	35,5	43,0
K	12					↓	3,19	3,59	4,22	5,72	7,64	10,7	14,9	18,1	23,9	29,3	34,4
L	15				↓	1,88	2,12	2,51	3,45	4,67	6,63	9,36	11,5	15,4	10,1	22,6	↑
M	18			↓	1,15	1,30	1,55	2,14	2,93	4,21	6,00	7,41	10,1	12,6	15,0	↑	
N	21		↓	0,698	0,795	0,945	1,32	1,81	2,63	3,77	4,68	6,41	8,06	9,64	↑		
P	25	↓	0,422	0,482	0,576	0,809	1,12	1,63	2,36	2,95	4,07	5,14	6,17	↑			
Q	32	0,244	0,279	0,336	0,475	0,662	0,973	1,42	1,78	2,47	3,14	3,80	↑				
R	40	0,163	0,197	0,279	0,391	0,580	0,851	1,07	1,50	1,92	2,32	↑					

NOTE The consumer's risk quality is the process fraction nonconforming at which 10 % of lots will be expected to be accepted.

Annex L
(informative)

Producer's risks

L.1 The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL, i.e. 1 minus the probability of accepting a given lot when the process fraction nonconforming is equal to the AQL.

L.2 For the "s" method, the producer's risk is given by the formula $F_{n-1,\sqrt{n}K_p}(\sqrt{n}k)$ where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the "s" method acceptability constant, K_p is the $(1-p)$ fractile of the standard normal distribution and $F_{n-1,\sqrt{n}K_p}(.)$ is the distribution function of the non-central t-distribution with degrees of freedom n–1 and non-centrality parameter $\sqrt{n}K_p$.

L.3 Producer's risks for the "s" method plans of this part of ISO 3951 are given in the following Tables L.1, L.3 and L.5 for normal, tightened and reduced inspections respectively.

L.4 For the "σ" method, the producer's risk is given by the formula $\Phi\{\sqrt{n}(k-K_p)\}$ where n is the sample size, p is the AQL expressed as a fraction nonconforming, k is the "σ" method acceptability constant, K_p is the $(1-p)$ fractile of the standard normal distribution and Φ (.) is the distribution function of the standard normal distribution.

L.5 Producer's risks for the "σ" method plans of this part of ISO 3951 are given in Tables L.2, L.4 and L.6 for normal, tightened and reduced inspections, respectively.

Table L.1 — Producer's risk (in percent) for normal inspection: "s" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	3													↓	10,9	12,8	9,7
C	4												↓	9,7	11,3	11,4	8,7
D	6											↓	8,1	9,6	9,7	9,3	5,3
E	9										↓	7,4	7,6	8,1	7,8	5,8	2,7
F	13									↓	7,0	7,4	6,7	7,3	5,7	3,9	1,3
G	18								↓	6,1	7,0	6,6	6,0	5,4	3,9	2,1	1,6
H	25							↓	5,3	6,0	6,3	6,1	4,4	3,8	2,3	2,6	1,9
J	35						↓	4,3	5,2	5,3	5,9	4,7	3,1	2,3	2,7	3,2	3,2
K	50					↓	3,7	3,6	4,1	4,4	4,0	3,0	1,5	2,3	2,7	4,0	2,4
L	70				↓	3,2	3,4	3,0	3,7	3,2	2,9	1,7	1,7	2,6	3,9	4,0	↑
M	95			↓	2,3	2,9	2,7	2,5	2,5	2,2	1,6	2,0	1,9	3,6	3,6	↑	
N	125		↓	1,8	2,2	2,5	2,5	1,7	1,8	1,3	2,1	2,5	2,8	3,7	↑		
P	160	↓	1,3	1,8	2,0	2,5	1,9	1,3	1,1	1,8	3,0	4,1	3,0	↑			
Q	200	1,2	1,2	1,5	1,8	1,7	1,3	0,7	1,5	2,3	4,4	4,0	↑				
R	250	1,2	1,0	1,5	1,4	1,4	0,9	1,1	2,2	3,9	5,2	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.2 — Producer's risk (in percent) for normal inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	2													↓	5,5	7,1	7,7
C	3												↓	4,7	5,8	6,6	6,6
D	4											↓	4,0	5,2	5,8	6,6	5,4
E	6										↓	3,3	3,5	4,2	4,7	4,4	3,2
F	8									↓	3,0	3,4	3,2	4,1	3,7	3,5	2,1
G	10								↓	2,5	3,2	3,2	3,1	3,3	3,0	2,5	2,8
H	12							↓	2,2	2,7	3,2	3,4	2,7	2,9	2,4	3,5	3,9
J	15						↓	1,5	2,2	2,6	3,3	2,8	2,2	2,2	3,2	4,7	5,9
K	18					↓	1,5	1,5	2,1	2,6	2,7	2,3	1,6	2,8	4,1	6,8	6,2
L	21				↓	1,4	1,7	1,6	2,4	2,3	2,5	2,0	2,4	4,2	6,6	8,2	↑
M	25			↓	1,0	1,5	1,6	1,6	1,9	2,0	1,9	2,7	3,1	6,1	7,2	↑	
N	32		↓	0,6	0,9	1,3	1,4	1,1	1,4	1,3	2,5	3,3	4,3	6,3	↑		
P	40	↓	0,3	0,6	0,8	1,3	1,1	0,8	0,9	1,8	3,3	5,1	4,7	↑			
Q	50	0,2	0,3	0,5	0,7	0,8	0,7	0,4	1,2	2,2	4,7	5,0	↑				
R	65	0,2	0,2	0,4	0,4	0,5	0,4	0,6	1,5	3,4	5,1	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.3 — Producer's risk (in percent) for tightened inspection: "s" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	3														↓	17,8	20,8
C	4													↓	15,7	19,5	20,3
D	6												↓	14,4	17,1	19,3	19,5
E	9											↓	12,3	15,2	16,6	18,6	15,8
F	13										↓	12,5	13,4	15,5	17,2	17,0	14,0
G	18									↓	12,2	13,5	13,4	16,0	15,6	15,4	11,0
H	25								↓	11,2	13,3	13,8	13,8	14,8	14,3	12,9	8,3
J	35							↓	10,4	12,2	13,8	14,7	12,7	13,8	12,3	10,7	9,1
K	50						↓	8,3	10,6	11,7	13,9	12,8	10,8	10,8	9,0	10,2	8,4
L	70					↓	8,3	8,7	10,6	12,5	13,1	12,0	8,8	8,8	9,7	11,5	↑
M	95				↓	7,8	8,7	8,4	11,1	11,2	12,0	9,8	6,6	9,0	10,1	↑	
N	125			↓	6,6	8,5	8,8	8,9	10,1	10,4	10,4	8,1	7,1	10,0	↑		
P	160		↓	5,9	7,4	9,0	10,0	8,4	9,9	9,4	9,4	9,5	8,3	↑			
Q	200	↓	4,4	6,4	7,5	9,8	8,9	7,5	8,2	7,7	10,1	10,3	↑				
R	250	4,7	5,0	6,8	8,8	9,5	8,9	6,6	7,4	9,3	12,5	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.4 — Producer's risk (in percent) for tightened inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	2														↓	10,3	12,8
C	3													↓	9,4	12,2	13,4
D	4												↓	9,2	11,4	13,6	14,8
E	6											↓	7,3	9,8	11,3	13,7	12,8
F	8										↓	7,6	8,4	10,5	12,5	13,3	12,3
G	10									↓	7,6	8,7	8,8	11,5	11,9	12,9	11,0
H	12								↓	6,9	9,0	9,6	10,0	11,5	12,0	12,3	10,1
J	15							↓	6,2	8,0	9,8	10,9	9,7	11,5	11,4	12,5	11,8
K	18						↓	4,8	7,0	8,4	10,9	10,3	9,2	10,3	10,0	12,7	12,9
L	21					↓	5,3	5,9	8,0	10,1	11,2	10,9	8,9	10,1	12,3	15,5	↑
M	25				↓	5,2	6,2	6,3	9,1	9,8	11,2	10,0	7,9	11,4	13,9	↑	
N	32			↓	3,9	5,7	6,3	6,8	8,4	9,4	10,1	8,8	8,6	12,7	↑		
P	40		↓	3,1	4,6	6,3	7,4	6,4	8,4	8,6	9,2	10,4	10,0	↑			
Q	50	↓	1,8	3,4	4,5	6,8	6,4	5,5	6,8	7,0	10,1	11,2	↑				
R	65	1,6	1,9	3,4	5,1	6,2	6,0	4,5	5,7	8,0	11,8	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.5 — Producer's risk (in percent) for reduced inspection: "s" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	3											↓	3,8	5,6	7,1	4,8	1,4
E	4										↓	3,5	4,7	6,4	5,6	3,8	0,9
F	6									↓	2,9	4,0	4,9	4,7	3,7	1,7	0,3
G	9								↓	2,1	3,2	4,1	3,4	3,0	1,6	0,5	0,4
H	13							↓	1,6	2,6	3,7	3,2	2,5	1,6	0,7	0,8	0,2
J	18						↓	1,2	2,1	3,0	2,9	2,4	1,4	0,8	1,0	0,5	0,5
K	25					↓	1,0	1,5	2,5	2,3	2,3	1,4	0,7	1,1	0,7	1,0	1,9
L	35				↓	0,8	1,3	1,8	1,8	1,8	1,3	0,8	0,9	0,8	1,2	3,2	↑
M	50			↓	0,4	0,9	1,4	1,1	1,2	0,8	0,6	0,8	0,5	1,0	2,7	↑	
N	70		↓	0,2	0,5	1,1	0,9	0,7	0,6	0,4	0,8	0,6	0,7	2,6	↑		
P	95	↓	0,1	0,3	0,7	0,7	0,6	0,3	0,3	0,5	0,5	0,8	1,9	↑			
Q	125	0,1	0,2	0,4	0,4	0,5	0,3	0,1	0,4	0,4	0,9	2,5	↑				
R	160	0,1	0,3	0,3	0,3	0,3	0,2	0,3	0,3	0,8	3,0	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Table L.6 — Producer's risk (in percent) for reduced inspection: "σ" method

Code letter	Sample size	Acceptance quality limit % nonconforming															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	2											↓	1,4	2,4	3,6	4,0	3,6
E	3										↓	1,0	1,6	2,6	2,8	2,8	1,8
F	4									↓	0,9	1,5	2,0	2,3	2,4	1,9	1,1
G	6								↓	0,4	0,9	1,4	1,3	1,4	1,1	0,8	0,9
H	8							↓	0,3	0,6	1,2	1,1	1,0	0,9	0,6	1,1	0,8
J	10						↓	0,2	0,4	0,8	0,9	0,9	0,6	0,6	1,0	1,0	1,3
K	12					↓	0,1	0,3	0,7	0,7	0,9	0,7	0,4	1,0	1,0	1,9	3,9
L	15				↓	0,09	0,2	0,4	0,5	0,6	0,6	0,4	0,7	0,9	1,7	4,7	↑
M	18			↓	0,05	0,2	0,4	0,3	0,5	0,4	0,4	0,7	0,6	1,5	4,1	↑	
N	21		↓	0,03	0,1	0,3	0,3	0,3	0,3	0,3	0,8	0,8	1,3	4,2	↑		
P	25	↓	0,01	0,07	0,2	0,3	0,3	0,2	0,2	0,6	0,8	1,4	3,1	↑			
Q	32	0,01	0,02	0,1	0,1	0,2	0,1	0,1	0,3	0,5	1,2	3,3	↑				
R	40	0,01	0,04	0,07	0,1	0,1	0,08	0,2	0,3	0,8	3,3	↑					

NOTE The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.

Annex M
(informative)

Operating characteristics for the "σ" method

M.1 Formula for probability of acceptance

The exact probability of lot acceptance for a single specification limit at process fraction nonconforming p is given by the formula

$$P_a = \Phi\{\sqrt{n}(K_p - k)\}$$

where $\Phi(.)$ denotes the standard normal distribution function, n is the sample size, K_p denotes the $(1 - p)$ fractile of the standard normal distribution and k is the "σ" method acceptance constant.

M.2 Example

Consider the calculation of the probability of acceptance at a process quality of 2,5 % nonconforming for a "σ" method plan with AQL 1,0 % and sample-size code letter M under normal inspection. Entering Table C.1 with sample-size code letter M and AQL 1,0 %, it is found that the sample size n is 25 and the acceptance constant k is 1,941. The process fraction nonconforming under consideration is p = 0,025, and from tables of the standard normal distribution it is found that K_p = 1,960. Hence,

$$P_\text{a} = \Phi\{\sqrt{25}(1{,}960 - 1{,}941)\} = \Phi(5 \times 0{,}019) = \Phi(0{,}095)$$

which, from standard normal distribution tables, yields P_a = 0,538.

M.3 Comparison with tabulated value for the "s" method

It is instructive to observe that this probability of acceptance for the "σ" method is very roughly in agreement with the corresponding probability of acceptance for the "s" method. From the column of the table in Chart M for AQL 1,0 %, it is seen that a process quality level of 2,51 %, i.e. p = 0,0251, corresponds to a probability of acceptance of 50 %, i.e. to P_a = 0,50.

Annex N
(informative)

Estimating the process fraction nonconforming for sample sizes 3 and 4 — "s" method

N.1 General formula for sample size n

The general formula for the estimator of the process fraction nonconforming beyond either of the specification limits when the process standard deviation is unknown is

$$\hat{p} = B_{(n-2)/2}\left\{\frac{1}{2}\left(1 - Q\frac{\sqrt{n}}{n-1}\right)\right\} \tag{N.1}$$

where n is the sample size, Q is the quality statistic and $B_{(n-2)/2}$ (.) is the symmetric beta distribution function with both parameters equal to (n–2)/2.

N.2 Formula for sample size 3

When n = 3, the estimator becomes

$$\hat{p} = B_{1/2}\left\{\frac{1}{2}\left(1 - Q\frac{\sqrt{3}}{2}\right)\right\} \tag{N.2}$$

Now

$$B_{\frac{1}{2}}(x) = \begin{cases} 0 & \text{if } x < 0, \\ \int_0^x \dfrac{t^{-\frac{1}{2}}(1-t)^{-\frac{1}{2}}}{B(\frac{1}{2},\frac{1}{2})}\,\mathrm{d}t & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } x > 1. \end{cases} \tag{N.3}$$

where

$$B(\tfrac{1}{2},\tfrac{1}{2}) = \Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{2})/\Gamma(\tfrac{1}{2}+\tfrac{1}{2}) = \sqrt{\pi}\sqrt{\pi}/1 = \pi,$$

Γ(.) representing the gamma function. Writing $t = \sin^2\theta$, Equation (N.3) becomes

$$B_{\frac{1}{2}}(x) = \begin{cases} 0 \\ \dfrac{2}{\pi}\displaystyle\int_0^{\arcsin(\sqrt{x})} d\theta \\ 1 \end{cases} = \begin{cases} 0 & \text{if } x < 0, \\ \dfrac{2}{\pi}\arcsin(\sqrt{x}) & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } x > 1. \end{cases} \tag{N.4}$$

Hence, substituting Equation (N.4) in Equation (N.2),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3}, \\ \dfrac{2}{\pi}\arcsin\left\{\sqrt{(1 - Q\sqrt{3}/2)/2}\right\} & \text{if } -2/\sqrt{3} \leqslant Q \leqslant 2/\sqrt{3}, \\ 1 & \text{if } Q < -2/\sqrt{3}. \end{cases}$$

This is the quantity tabulated in Annex F.

N.3 Formula for sample size 4

When n = 4, the estimator becomes

$$\hat{p} = B_1\left\{\tfrac{1}{2}\left(1 - \tfrac{2}{3}Q\right)\right\} = B_1\{0{,}5 - Q/3\}. \qquad \text{(N.5)}$$

Now

$$B_1(x) = \begin{cases} 0 & \text{if } x < 0, \\ \displaystyle\int_0^x \frac{dt}{B(1,1)} & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } x > 1, \end{cases} \qquad \text{(N.6)}$$

where $B(1,1) = \Gamma(1)\Gamma(1)/\Gamma(1 + 1) = 1$.

Equation (N.6) can therefore be written

$$B_1(x) = \begin{cases} 0 & \text{if } x < 0, \\ x & \text{if } 0 \leqslant x \leqslant 1, \\ 1 & \text{if } x > 1. \end{cases} \qquad \text{(N.7)}$$

Hence, substituting Equation (N.7) in Equation (N.5),

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 1{,}5, \\ 0{,}5 - Q/3 & \text{if } -1{,}5 \leqslant Q \leqslant 1{,}5, \\ 1 & \text{if } Q < -1{,}5. \end{cases}$$

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ICS 03.120.30

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